Complete formulas used by `coverage`

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The `coverage` function is used to define a new function to compute the probability density function (pdf) of cumulative distribution function (cdf) of any probability law[1] under the following insurance coverage modifications: ordinary or franchise deductible, limit, coinsurance, inflation. In addition, the function can return the distribution of either the payment per loss or the payment per payment random variable. For the exact definitions of these terms as used by `coverage`, see Chapter 5 of [1].

In the presence of a deductible, four random variables can be defined:

1. Y^P, the payment per payment with an ordinary deductible;
2. Y^L, the payment per loss with an ordinary deductible;
3. $\tilde{Y}^P$, the payment per payment with a franchise deductible;
4. $\tilde{Y}^L$, the payment per loss with a franchise deductible.

The most common case in insurance applications is the distribution of the amount paid per payment with an ordinary deductible, Y^P. Hence, it is the default in `coverage`.

When there is no deductible, all four random variables are equivalent.

This vignette presents the definitions of the above four random variables and their corresponding cdf and pdf for a deductible d, a limit u, a coinsurance level α and an inflation rate r. An illustrative plot of each cdf and pdf is also included. In these plots, a dot at the end of a vertical bar represents a probability mass at the given point.

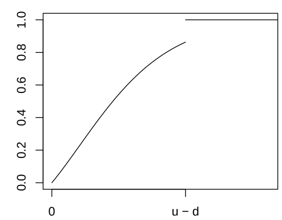
In definitions below, X is the nonnegative random variable of the losses with cdf $F_X(\cdot)$ and pdf $f_X(\cdot)$.

[1] Provided functions `pfoo` and `dfoo` exist in the current R frame for probability law `foo`.

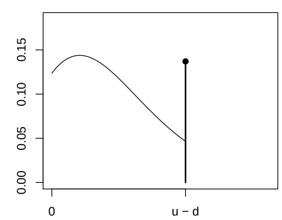
1 Payment per payment, ordinary deductible

$$Y^P = \begin{cases} \text{undefined}, & X < \dfrac{d}{1+r} \\ \alpha((1+r)X - d), & \dfrac{d}{1+r} \leq X < \dfrac{u}{1+r} \\ \alpha(u-d), & X \geq \dfrac{u}{1+r} \end{cases}$$

$$F_{Y^P}(y) = \begin{cases} 0, & y = 0 \\ \dfrac{F_X\left(\frac{y+\alpha d}{\alpha(1+r)}\right) - F_X\left(\frac{d}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & 0 < y < \alpha(u-d) \\ 1, & y \geq \alpha(u-d) \end{cases}$$



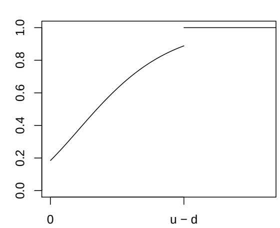
$$f_{Y^P}(y) = \begin{cases} 0, & y = 0 \\ \left(\dfrac{1}{\alpha(1+r)}\right) \dfrac{f_X\left(\frac{y+\alpha d}{\alpha(1+r)}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & 0 < y < \alpha(u-d) \\ \dfrac{1 - F_X\left(\frac{u}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & y = \alpha(u-d) \end{cases}$$



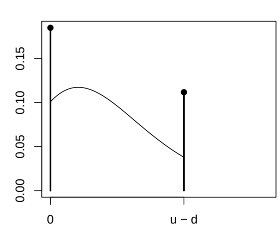
2 Payment per loss, ordinary deductible

$$Y^L = \begin{cases} 0, & X < \dfrac{d}{1+r} \\ \alpha((1+r)X - d), & \dfrac{d}{1+r} \leq X < \dfrac{u}{1+r} \\ \alpha(u-d), & X \geq \dfrac{u}{1+r} \end{cases}$$

$$F_{Y^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & y = 0 \\ F_X\left(\dfrac{y+\alpha d}{\alpha(1+r)}\right), & 0 < y < \alpha(u-d) \\ 1, & y \geq \alpha(u-d) \end{cases}$$



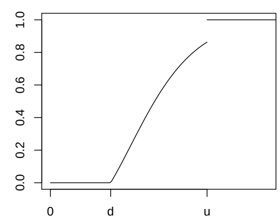
$$f_{Y^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & y = 0 \\ \dfrac{1}{\alpha(1+r)} f_X\left(\dfrac{y+\alpha d}{\alpha(1+r)}\right), & 0 < y < \alpha(u-d) \\ 1 - F_X\left(\dfrac{u}{1+r}\right), & y = \alpha(u-d) \end{cases}$$



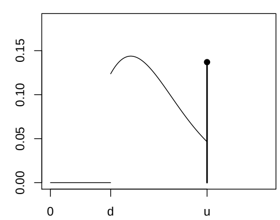
3 Payment per payment, franchise deductible

$$\tilde{Y}^P = \begin{cases} \text{undefined}, & X < \dfrac{d}{1+r} \\ \alpha(1+r)X, & \dfrac{d}{1+r} \leq X < \dfrac{u}{1+r} \\ \alpha u, & X \geq \dfrac{u}{1+r} \end{cases}$$

$$F_{\tilde{Y}^P}(y) = \begin{cases} 0, & 0 \leq y \leq \alpha d \\ \dfrac{F_X\left(\frac{y}{\alpha(1+r)}\right) - F_X\left(\frac{d}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & \alpha d < y < \alpha u \\ 1, & y \geq \alpha u \end{cases}$$



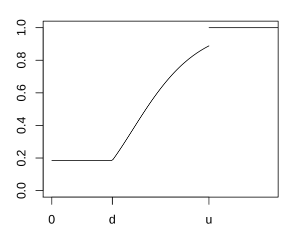
$$f_{\tilde{Y}^P}(y) = \begin{cases} 0, & 0 \leq y \leq \alpha d \\ \left(\dfrac{1}{\alpha(1+r)}\right) \dfrac{f_X\left(\frac{y}{\alpha(1+r)}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & \alpha d < y < \alpha u \\ \dfrac{1 - F_X\left(\frac{u}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)}, & y = \alpha u \end{cases}$$



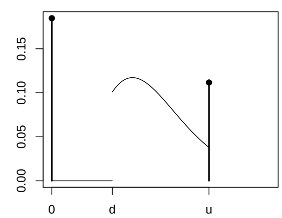
4 Payment per loss, franchise deductible

$$\tilde{Y}^L = \begin{cases} 0, & X < \dfrac{d}{1+r} \\ \alpha(1+r)X, & \dfrac{d}{1+r} \leq X < \dfrac{u}{1+r} \\ \alpha u, & X \geq \dfrac{u}{1+r} \end{cases}$$

$$F_{\tilde{Y}^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & 0 \leq y \leq \alpha d \\ F_X\left(\dfrac{y}{\alpha(1+r)}\right), & \alpha d < y < \alpha u \\ 1, & y \geq \alpha u \end{cases}$$



$$f_{\tilde{Y}^L}(y) = \begin{cases} F_X\left(\dfrac{d}{1+r}\right), & y = 0 \\ \dfrac{1}{\alpha(1+r)} f_X\left(\dfrac{y}{\alpha(1+r)}\right), & \alpha d < y < \alpha u \\ 1 - F_X\left(\dfrac{u}{1+r}\right), & y = \alpha u \end{cases}$$



References

[1] S. A. Klugman, H. H. Panjer, and G. Willmot. *Loss Models: From Data to Decisions.* Wiley, New York, 2 edition, 2004.