

index.G1(clusterSim)

### Caliński-Harabasz pseudo F-Statistic

$$G1(u) = \frac{\text{trace}(\mathbf{B}_u)/(u-1)}{\text{trace}(\mathbf{W}_u)/(n-u)},$$

where:  $\mathbf{X} = \{x_{ij}\}$ ,  $i = 1, \dots, n$ ;  $j = 1, \dots, m$  – data matrix,

$n$  – number of objects,

$m$  – number of variables,

$u$  – number of clusters ( $u = 2, \dots, n-1$ ),

$\mathbf{W}_u = \sum_r \sum_{i \in C_r} (\mathbf{x}_{ri} - \bar{\mathbf{x}}_r)(\mathbf{x}_{ri} - \bar{\mathbf{x}}_r)^T$  – within-group dispersion matrix for data clustered into  $u$

clusters,

$\mathbf{B}_u = \sum_r n_r (\bar{\mathbf{x}}_r - \bar{\mathbf{x}})(\bar{\mathbf{x}}_r - \bar{\mathbf{x}})^T$  – between-group dispersion matrix for data clustered into  $u$

clusters,

$r = 1, \dots, u$  – cluster number,

$\bar{\mathbf{x}}_r$  – centroid or medoid of cluster  $r$ ,

$\bar{\mathbf{x}}$  – centroid or medoid of data matrix,

$C_r$  – the indices of objects in cluster  $r$ ,

$n_r$  – number of objects in cluster  $r$ .

The value of  $u$ , which maximizes  $G1(u)$ , is regarded as specifying the number of clusters.

### References

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