Package ‘MRCE’

January 5, 2017

Type Package

Title Multivariate Regression with Covariance Estimation

Version 2.1

Date 2017-01-04

Author Adam J. Rothman

Maintainer Adam J. Rothman <arothman@umn.edu>

Depends R (>= 2.10.1), QUIC

Description Compute and select tuning parameters for the MRCE estimator proposed by Rothman, Levina, and Zhu (2010) <doi:10.1198/jcgs.2010.09188>. This estimator fits the multiple output linear regression model with a sparse estimator of the error precision matrix and a sparse estimator of the regression coefficient matrix.

License GPL-2

NeedsCompilation yes

Repository CRAN

Date/Publication 2017-01-05 10:39:24

R topics documented:

`MRCE-package` *Multivariate regression with covariance estimation*

Description

Computes the MRCE estimators (Rothman, Levina, and Zhu, 2010) and has the dataset `stock04` used in Rothman, Levina, and Zhu (2010), originally analyzed in Yuan et al. (2007).

Details

The primary function is `mrce`. The dataset is `stock04`.

Author(s)

Adam J. Rothman

Maintainer: Adam J. Rothman <arothman@umn.edu>

References

Rothman, A.J., Levina, E., and Zhu, J. (2010). Sparse multivariate regression with covariance estimation. Journal of Computational and Graphical Statistics 19:974–962.

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. Journal of the Royal Statistical Society Series B 69(3):329–346.

Cho-Jui Hsieh, Matyas A. Sustik, Inderjit S. Dhillon, and Pradeep Ravikumar (2011). Sparse inverse covariance matrix estimation using quadratic approximation. Advances in Neural Information Processing Systems 24, 2011, p. 2330–2338.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. Journal of Statistical Software, 33(1), 1-22.

`mrce` *Do multivariate regression with covariance estimation (MRCE)*

Description

Let S^q_+ be the set of q by q symmetric and positive definite matrices and let $y_i \in R^q$ be the measurements of the q responses for the ith case/observation ($i = 1, \ldots, n$). The model assumes that $y_i \in R^q$ is a realization of the q-variate random vector

$$Y_i = \mu + B_0' x_i + E_i, \quad i = 1, \ldots, n$$

where $\mu \in R^q$ is an unknown intercept vector, $B_0 \in R^{p \times q}$ is an unknown regression coefficient matrix, $x_i \in R^p$ is a known vector of values for ith case's p predictors, and $E_1, \ldots, E_n$ are n independent copies of a q-variate Normal random vector with mean 0 and unknown inverse covariance matrix $\Omega_0 \in S^q_+$.

This function computes penalized likelihood estimates of the unknown parameters μ, B_0, and Ω_0. Let $\bar{y} = n^{-1}\sum_{i=1}^n y_i$ and $\bar{x} = n^{-1}\sum_{i=1}^n x_i$. These estimates are

$$(\hat{B}, \hat{\Omega}) = \arg \min_{B \in R^{p\times q}, \Omega \in S^q_+} \left\{ g(B, \Omega) + \lambda_1 \sum_{j \neq k} |\omega_{jk}| + 2\lambda_2 \sum_{j=1}^p \sum_{k=1}^q |b_{jk}| \right\}$$

and $\hat{\mu} = \bar{y} - \hat{B}'\bar{x}$, where

$$g(B, \Omega) = \mathrm{tr}\{n^{-1}(Y - XB)'(Y - XB)\Omega\} - \log|\Omega|,$$

$Y \in R^{n \times q}$ has ith row $(y_i - \bar{y})'$, and $X \in R^{n \times p}$ has ith row $(x_i - \bar{x})'$.

Usage

```
mrce(X,Y, lam1=NULL, lam2=NULL, lam1.vec=NULL, lam2.vec=NULL,
     method=c("single", "cv", "fixed.omega"),
     cov.tol=1e-4, cov.maxit=1e3, omega=NULL,
     maxit.out=1e3, maxit.in=1e3, tol.out=1e-8,
     tol.in=1e-8, kfold=5, silent=TRUE, eps=1e-5)
```

Arguments

X	An n by p matrix of the values for the prediction variables. Do not center its columns, the ith row of X is x_i defined above ($i = 1, \ldots, n$). Do not include a column of ones.
Y	An n by q matrix of the observed responses. Do not center its columns, the ith row of Y is y_i defined above ($i = 1, \ldots, n$).
lam1	A single value for λ_1 defined above. This argument is only used if method="single"
lam2	A single value for λ_2 defined above (or a p by q matrix with (j, k)th entry λ_{2jk} in which case the penalty $2\lambda_2 \sum_{j=1}^{p} \sum_{k=1}^{q} \lvert b_{jk} \rvert$ becomes $2 \sum_{j=1}^{p} \sum_{k=1}^{q} \lambda_{2jk} \lvert b_{jk} \rvert$). This argument is not used if method="cv".
lam1.vec	A vector of candidate values for λ_1 from which the cross validation procedure searches: only used when method="cv" and must be specified by the user when method="cv". Please arrange in decreasing order.
lam2.vec	A vector of candidate values for λ_2 from which the cross validation procedure searches: only used when method="cv" and must be specified by the user when method="cv". Please arrange in decreasing order.
method	There are three options: • method="single" computes the MRCE estimate of the regression coefficient matrix with penalty tuning parameters lam1 and lam2; • method="cv" performs kfold cross validation using candidate tuning parameters in lam1.vec and lam2.vec; • method="fixed.omega" computes the regression coefficient matrix estimate for which Ω (defined above) is fixed at omega.
cov.tol	Convergence tolerance for the QUIC algorithm that minimizes the objective function (defined above) with B fixed.
cov.maxit	The maximum number of iterations allowed for the QUIC algorithm that minimizes the objective function (defined above) with B fixed.
omega	A user-supplied fixed value of Ω. Only used when method="fixed.omega" in which case the minimizer of the objective function (defined above) with Ω fixed at omega is returned.
maxit.out	The maximum number of iterations allowed for the outer loop of the exact MRCE algorithm.
maxit.in	The maximum number of iterations allowed for the algorithm that minimizes the objective function, defined above, with Ω fixed.
tol.out	Convergence tolerance for outer loop of the exact MRCE algorithm.

`tol.in`	Convergence tolerance for the algorithm that minimizes the objective function, defined above, with Ω fixed.
`kfold`	The number of folds to use when `method="cv"`.
`silent`	Logical: when `silent=FALSE` this function displays progress updates to the screen.
`eps`	The algorithm will terminate if the minimum diagonal entry of the current iterate's residual sample covariance is less than `eps`. This may need adjustment depending on the scales of the variables.

Details

Please see Rothman, Levina, and Zhu (2010) for more information on the algorithm and model. This version of the software uses the QUIC algorithm (Hsieh et al., 2011) through the R package `QUIC`. If the algorithm is running slowly, track its progress with `silent=FALSE`. In some cases, choosing `cov.tol=0.1` and `tol.out=1e-10` allows the algorithm to make faster progress. If one uses a matrix for `lam2`, consider setting `tol.in=1e-12`.

When $p > n$ and λ_2 is too close to zero, a perfect fit will occur. The algorithm will terminate early and inform the user.

The algorithm that minimizes the objective function, defined above, with Ω fixed uses a similar update strategy and termination criterion to those used by Friedman et al. (2010) in the corresponding R package `glmnet`.

Value

A list containing

`Bhat`	This is $\hat{B} \in R^{p\times q}$ defined above. If `method="cv"`, then `best.lam1` and `best.lam2` defined below are used for λ_1 and λ_2.
`muhat`	This is the intercept estimate $\hat{\mu} \in R^q$ defined above. If `method="cv"`, then `best.lam1` and `best.lam2` defined below are used for λ_1 and λ_2.
`omega`	This is $\hat{\Omega} \in S_+^q$ defined above. If `method="cv"`, then `best.lam1` and `best.lam2` defined below are used for λ_1 and λ_2.
`mx`	This is $\bar{x} \in R^p$ defined above.
`my`	This is $\bar{y} \in R^q$ defined above.
`best.lam1`	The selected value for λ_1 by cross validation. Will be `NULL` unless `method="cv"`.
`best.lam2`	The selected value for λ_2 by cross validation. Will be `NULL` unless `method="cv"`.
`cv.err`	Cross validation error matrix with `length(lam1.vec)` rows and `length(lam2.vec)` columns. Will be `NULL` unless `method="cv"`.

Note

The algorithm is fastest when λ_1 and λ_2 are large. Please do not set these parameters to zero. Use `silent=FALSE` to check if the algorithm is converging before the total iterations exceeds `maxit.out`.

Author(s)

Adam J. Rothman

References

Rothman, A. J., Levina, E., and Zhu, J. (2010) Sparse multivariate regression with covariance estimation. Journal of Computational and Graphical Statistics. 19: 947–962.

Cho-Jui Hsieh, Matyas A. Sustik, Inderjit S. Dhillon, and Pradeep Ravikumar (2011). Sparse inverse covariance matrix estimation using quadratic approximation. Advances in Neural Information Processing Systems 24, 2011, p. 2330–2338.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. Journal of Statistical Software, 33(1), 1-22.

Examples

```
set.seed(5)
n=50
p=10
q=5

Omega0.inv=diag(q)
for(i in 1:q) for(j in 1:q)
  Omega0.inv[i,j]=0.7^abs(i-j)
out=eigen(Omega0.inv, symmetric=TRUE)
Omega0.inv.sqrt=tcrossprod(out$vec*rep(out$val^(0.5), each=q),out$vec)
Omega0=tcrossprod(out$vec*rep(out$val^(-1), each=q),out$vec)

X=matrix(rnorm(n*p), nrow=n, ncol=p)
E=matrix(rnorm(n*p), nrow=n, ncol=q)%*%Omega0.inv.sqrt
B0=matrix(rbinom(p*q, size=1, prob=0.1)*runif(p*q, min=1, max=2), nrow=p, ncol=q)

Y=X%*%B0 + E

lam1.vec=rev(10^seq(from=-2, to=0, by=0.5))
lam2.vec=rev(10^seq(from=-2, to=0, by=0.5))
cvfit=mrce(Y=Y, X=X, lam1.vec=lam1.vec, lam2.vec=lam2.vec, method="cv")
cvfit

fit=mrce(Y=Y, X=X, lam1=10^(-1.5), lam2=10^(-0.5), method="single")
fit

lam2.mat=1000*(fit$Bhat==0)
refit=mrce(Y=Y, X=X, lam2=lam2.mat, method="fixed.omega", omega=fit$omega, tol.in=1e-12)
refit
```

stock04 *log-returns of 9 stocks from 2004*

Description

Weekly log-returns of 9 stocks from 2004, analyzed in Yuan et al. (2007)

Usage

```
data(stock04)
```

Format

The format is: num [1:52, 1:9] 0.002275 -0.003795 0.012845 0.017489 -0.000369 ... - attr(*, "dimnames")=List of 2 ..$: NULL ..$: chr [1:9] "Walmart" "Exxon" "GM" "Ford" ...

Source

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. Journal of the Royal Statistical Society Series B, 69(3):329–346.

References

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. Journal of the Royal Statistical Society Series B, 69(3):329–346.

Index