

# Step-by-step integration

This is another nice example drawn from the Pythontex gallery, see <https://github.com/gpoore/pythontex>.

It shows the step-by-step computations of a simple triple integral.

```
from sympy import *

x, y, z = symbols('x,y,z')
f = Symbol('f(x,y,z)')

# Define limits of integration
x_max = 2;   y_max = 3;   z_max = 4;
x_min = 0;   y_min = 0;   z_min = 0;

lhs = Integral(f, (x, x_min, x_max),
               (y, y_min, y_max),
               (z, z_min, z_max))           # py(lhs.01,lhs)

f = x*y + y*sin(z) + cos(x+y)

rhs = Integral(f, (x, x_min, x_max),
               (y, y_min, y_max),
               (z, z_min, z_max))         # py(rhs.01,rhs)
rhs = Integral(Integral(f, (x, x_min, x_max)).doit(),
               (y, y_min, y_max),
               (z, z_min, z_max))         # py(rhs.02,rhs)
rhs = Integral(Integral(f, (x, x_min, x_max),
                       (y, y_min, y_max)).doit(),
               (z, z_min, z_max))         # py(rhs.03,rhs)
rhs = Integral(f, (x, x_min, x_max),
               (y, y_min, y_max),
               (z, z_min, z_max)).doit()  # py(rhs.04,rhs)

# And now, a numerical approximation
rhs = N(rhs)                             # py(rhs.05,rhs)
```

$$\begin{aligned}
\int_0^4 \int_0^3 \int_0^2 f(x, y, z) dx dy dz &= \int_0^4 \int_0^3 \int_0^2 (xy + y \sin(z) + \cos(x + y)) dx dy dz \\
&= \int_0^4 \int_0^3 (2y \sin(z) + 2y - \sin(y) + \sin(y + 2)) dy dz \\
&= \int_0^4 (9 \sin(z) + \cos(3) + \cos(2) - \cos(5) + 8) dz \\
&= 4 \cos(3) + 4 \cos(2) - 4 \cos(5) - 9 \cos(4) + 41 \\
&\approx 40.1235865133293
\end{aligned}$$

```

\begin{align*}
\py{lhs.01} &= \py{rhs.01} \\
&= \py{rhs.02} \\
&= \py{rhs.03} \\
&= \py{rhs.04} [8pt] \\
&\approx \py{rhs.05}
\end{align*}

```