

# Package ‘MSTest’

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**Type** Package

**Title** Hypothesis Testing for Markov Switching Models

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**Description** Implementation of hypothesis testing procedures described in Hansen (1992) <doi:10.1002/jae.3950070506>, Carrasco, Hu, & Ploberger (2014) <doi:10.3982/ECTA8609>, Dufour & Luger (2017) <doi:10.1080/07474938.2017.1307548>, and Rodriguez Rondon & Dufour (2022) <[https://rodriguezrondon.com/files/RodriguezRondon\\_Dufour\\_MonteCarlo\\_LikelihoodRatioTest\\_MarkovSwitchingModels.pdf](https://rodriguezrondon.com/files/RodriguezRondon_Dufour_MonteCarlo_LikelihoodRatioTest_MarkovSwitchingModels.pdf)> that can be used to identify the number of regimes in Markov switching models.

**License** GPL (>= 2)

**LazyData** TRUE

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**URL** <https://github.com/roga11/MSTest>

**BugReports** <https://github.com/roga11/MSTest/issues>

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**LinkingTo** Rcpp, RcppArmadillo

**Depends** R (>= 4.0.0)

**SystemRequirements** C++17

**NeedsCompilation** yes

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MSTest-package	<i>Testing Markov Switching Models</i>
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**Description**

This package implements hypothesis testing procedures that can be used to identify the number of regimes in a Markov-Switching model.

**Author(s)**

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## References

- Carrasco, Marine, Liang Hu, and Werner Ploberger. 2014. "Optimal test for Markov switching parameters." *Econometrica* 82 (2): 765–784.
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- Rodriguez Rondon, Gabriel and Jean-Marie Dufour. 2022. "MSTest: An R-package for Testing Markov-Switching Models." *Unpublished manuscript*.
- Qu, Zhongjun, and Fan Zhuo. 2021. "Likelihood Ratio-Based Tests for Markov Regime Switching." *The Review of Economic Studies* 88 (2): 937–968.

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ARmdl

*Autoregressive Model*

---

## Description

This function estimates an autoregressive model with  $p$  lags. This can be used for the null hypothesis of a linear model against an alternative hypothesis of a Markov switching autoregressive model with  $k$  regimes.

## Usage

```
ARmdl(Y, p, control = list())
```

**Arguments**

Y	A ( $T \times 1$ ) matrix of observations.
p	Integer determining the number of autoregressive lags.
control	List with model options including: <ul style="list-style-type: none"> <li>• const: Boolean determining whether to estimate model with constant if TRUE or not if FALSE. Default is TRUE.</li> <li>• getSE: Boolean determining whether to compute standard errors of parameters if TRUE or not if FALSE. Default is TRUE.</li> </ul>

**Value**

List of class ARmdl (S3 object) with model attributes including:

- y: a ( $T-p \times 1$ ) matrix of observations.
- X: a ( $T-p \times p + \text{const}$ ) matrix of lagged observations with a leading column of 1s if const=TRUE or not if const=FALSE.
- x: a ( $T-p \times p$ ) matrix of lagged observations.
- fitted: a ( $T-p \times 1$ ) matrix of fitted values.
- resid: a ( $T-p \times 1$ ) matrix of residuals.
- inter: estimated intercept of the process.
- mu: estimated mean of the process.
- coef: coefficient estimates. First value is the intercept (i.e., not mu) if const=TRUE. This is the same as phi if const=FALSE.
- intercept: estimate of intercept.
- phi: estimates of autoregressive coefficients.
- stdev: estimated standard deviation of the process.
- sigma: estimated variance of the process.
- theta: vector containing: mu, sigma, and phi.
- theta\_mu\_ind: vector indicating location of mean with 1 and 0 otherwise.
- theta\_sig\_ind: vector indicating location of variance with 1 and 0 otherwise.
- theta\_var\_ind: vector indicating location of variance with 1 and 0 otherwise. This is the same as theta\_sig\_ind in ARmdl.
- theta\_phi\_ind: vector indicating location of autoregressive coefficients with 1 and 0 otherwise.
- stationary: Boolean indicating if process is stationary if TRUE or non-stationary if FALSE.
- n: number of observations after lag transformation (i.e.,  $n = T-p$ ).
- p: number of autoregressive lags.
- q: number of series. This is always 1 in ARmdl.
- k: number of regimes. This is always 1 in ARmdl.
- control: List with model options used.
- logLike: log-likelihood.

- AIC: Akaike information criterion.
- BIC: Bayesian (Schwarz) information criterion.
- Hess: Hessian matrix. Approximated using [hessian](#) and only returned if `getSE=TRUE`.
- `info_mat`: Information matrix. Computed as the inverse of `-Hess`. If matrix is not PD then nearest PD matrix is obtained using [nearest\\_spd](#). Only returned if `getSE=TRUE`.
- `nearPD_used`: Boolean determining whether `nearPD` function was used on `info_mat` if `TRUE` or not if `FALSE`. Only returned if `getSE=TRUE`.
- `theta_se`: standard errors of parameters in `theta`. Only returned if `getSE=TRUE`.

### See Also

[MSARmdl](#)

### Examples

```
set.seed(1234)
# Define DGP of AR process
mdl_ar <- list(n      = 500,
              mu      = 5,
              sigma    = 2,
              phi      = c(0.5,0.2))

# Simulate process using simuAR() function
y_simu <- simuAR(mdl_ar)

# Set options for model estimation
control <- list(const = TRUE,
                getSE = TRUE)

# Estimate model
y_ar_mdl <- ARmdl(y_simu$y, p = 2, control)
y_ar_mdl
```

---

BootLRTest

*Bootstrap Likelihood Ratio Test*

---

### Description

This function performs the bootstrap likelihood ratio test discussed in Qu & Zhuo (2021) and Kasahara & Shimotsu (2018).

### Usage

```
BootLRTest(Y, p, k0, k1, control = list())
```

## Arguments

Y	Series to be tested. Must be a (T x q) matrix.
p	Number of autoregressive lags. Must be greater than or equal to 0.
k0	Number of regimes under null hypothesis. Must be greater than or equal to 1.
k1	Number of regimes under alternative hypothesis. Must be greater than k0.
control	List with test procedure options including: <ul style="list-style-type: none"> <li>• B: Integer determining the number of bootstrap simulations. Default is set to 999.</li> <li>• burnin: Number of simulated observations to remove from beginning. Default is 100.</li> <li>• converge_check: String of NULL determining if convergence of model(s) should be verified. Allowed inputs are: "null", "alt", "both", or NULL. If NULL (default) no model convergence is verified.</li> <li>• workers: Integer determining the number of workers to use for parallel computing version of test. Note that parallel pool must already be open. Default is 0.</li> <li>• mdl_h0_control: List with restricted model options. See <a href="#">Nmdl</a>, <a href="#">ARmdl</a>, <a href="#">VARmdl</a>, <a href="#">HMmdl</a>, <a href="#">MSARmdl</a>, or <a href="#">MSVARmdl</a> documentation for available and default values.</li> <li>• mdl_h1_control: List with unrestricted model options. See <a href="#">HMmdl</a>, <a href="#">MSARmdl</a>, or <a href="#">MSVARmdl</a> documentation for available and default values.</li> </ul>

## Value

List of class LMCLRTTest (S3 object) with attributes including:

- mdl\_h0: List with restricted model attributes. See [Nmdl](#), [ARmdl](#), [VARmdl](#), [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for return values.
- mdl\_h1: List with unrestricted model attributes. See [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for return values.
- LRT\_0: Value of test statistic from observed data.
- LRN: A (N x 1) vector of test statistics from data simulated under the null hypothesis.
- pval: P-value of Local Monte Carlo Likelihood Ratio Test.
- LRN\_cv: Vector with 90%, 95%, and 99% Monte Carlo critical values (from vector LRN).
- control: List with test procedure options used.

## References

- Qu, Zhongjun, and Fan Zhuo. 2021. "Likelihood Ratio-Based Tests for Markov Regime Switching." *The Review of Economic Studies* 88 (2): 937–968.
- Kasahara, Hiroyuk, and Katsum Shimotsu. 2018. "Testing the number of regimes in Markov regime switching models." *arXiv preprint arXiv:1801.06862*.
- Rodriguez-Rondon, Gabriel and Jean-Marie Dufour. 2023. "Monte Carlo Likelihood Ratio Tests for Markov Switching Models." *Unpublished manuscript*.

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chp10GNP

*Carrasco, Hu, & Ploberger 2010 GNP data*

---

**Description**

Carrasco, Hu, & Ploberger 2010 GNP data

**Usage**

chp10GNP

**Format**

This data is the extension of the GNP series used in CHP (2014), *Econometrica*. This series ranges from 1951Q2 to 2010Q4.

Vector of dates

**DATE** log difference of US GNP series

**GNP** US GNP series

**Source**

<https://www.econometricsociety.org/content/supplement-optimal-test-markov-switching-parameters>

**References**

Carrasco, Marine, Liang Hu, and Werner Ploberger. 2014. "Optimal test for Markov switching parameters." *Econometrica* 82 (2): 765–784.

---

CHPTest

*Carrasco, Hu, and Ploberger (2014) parameter stability test*

---

**Description**

This function performs the CHP (2014) parameter stability test as outline in Carrasco, M., Hu, L. and Ploberger, W. (2014). Original source code can be found [here](#).

**Usage**

CHPTest(Y, p, control = list())

**Arguments**

Y	A (T x 1) matrix of observations.
p	Integer determining the number of autoregressive lags.
control	List with test procedure options including: <ul style="list-style-type: none"> <li>• N: Integer determining the number of Bootstrap iterations. Default is set to 3000 as in paper.</li> <li>• rho_b: Number determining bounds for distribution of <math>\rho</math> (i.e. <math>\rho \sim [-\rho_b, \rho_b]</math>).</li> <li>• msvar: Boolean indicator. If TRUE, there is a switch in variance. If FALSE only switch in mean is considered.</li> <li>• getSE: Boolean indicator. If TRUE, standard errors for restricted model are estimated. If FALSE no standard errors are estimated. Default is TRUE.</li> </ul>

**Value**

List of class CHPTest (S3 object) with model attributes including:

- mdl\_h0: List with restricted model attributes. This will be of class ARmdl (S3 object). See [ARmdl](#).
- supTS: supTS test statistic value.
- expTS: expTS test statistic value.
- supTS\_N: A (N x 1) vector with simulated supTS test statistics under null hypothesis.
- expTS\_N: A (N x 1) vector with simulated expTS test statistics under null hypothesis.
- pval\_supTS: P-value for supTS version of parameter stability test.
- pval\_expTS: P-value for expTS version of parameter stability test.
- supTS\_cv: Vector with 90%, 95%, and 99% bootstrap critical values for supTS version of parameter stability test.
- expTS\_cv: Vector with 90%, 95%, and 99% bootstrap critical values for expTS version of parameter stability test.
- control: List with test procedure options used.

**References**

Carrasco, Marine, Liang Hu, and Werner Ploberger. 2014. "Optimal test for Markov switching parameters." *Econometrica* 82 (2): 765–784.

**Examples**

```
# load data used in Hamilton 1989
y84 <- as.matrix(hamilton84GNP$GNP_logdiff)

# Set test procedure options
control = list(N = 1000,
              rho_b = 0.7,
```



```

msvar = FALSE)

# perform test with switch in mean only on Hamilton 1989 data

mdl_84_msmu <- CHPTest(y84, p = 4, control = control)
summary(mdl_84_msmu)

```

---

DLMCTest

*Monte Carlo moment-based test for Markov switching model*


---

### Description

This function performs the Local Monte Carlo moment-based test for Markov switching autoregressive models proposed in Dufour & Luger (2017).

### Usage

```
DLMCTest(Y, p, control = list())
```

### Arguments

Y	Series to be tested
p	Number of autoregressive lags.
control	List with test procedure options including: <ul style="list-style-type: none"> <li>• N: Integer determining the number of Monte Carlo simulations. Default is set to 99 as in paper.</li> <li>• simdist_N: Integer determining the number of simulations for CDF distribution approximation. Default is set to 10000.</li> <li>• getSE: Boolean indicator. If TRUE, standard errors for restricted model are estimated. If FALSE no standard errors are estimated. Default is TRUE.</li> </ul>

### Value

List of class DLMCTest (S3 object) with attributes including:

- mdl\_h0: List with restricted model attributes. This will be of class ARmdl if  $p > 0$  or Nmdl otherwise (S3 objects). See [ARmdl](#) or [Nmdl](#).
- theta: Value of nuisance parameters. Specifically, these are the consistent estimates of nuisance parameters as discussed in Dufour & Luger (2017) LMC procedure.
- S0: A (1 × 4) matrix containing the four moment-based test statistics defined in (11) - (14) in Dufour & Luger (2017).
- F0\_min: Test statistic value for min version of Local Monte Carlo moment-based test.
- F0\_prod: Test statistic value for prod version of Local Monte Carlo moment-based test.
- FN\_min: A (N × 1) vector with simulated test statistics for min version of Local Monte Carlo moment-based test under null hypothesis.

- FN\_prod: A ( $N \times 1$ ) vector with simulated test statistics for prod version of Local Monte Carlo moment-based test under null hypothesis.
- pval\_min: P-value for min version of Local Monte Carlo moment-based test.
- pval\_prod: P-value for prod version of Local Monte Carlo moment-based test.
- FN\_min\_cv: Vector with 90%, 95%, and 99% Monte Carlo critical values for min version of Local Monte Carlo moment-based test.
- FN\_prod\_cv: Vector with 90%, 95%, and 99% Monte Carlo critical values for prod version of Local Monte Carlo moment-based test.
- control: List with test procedure options used.

## References

Dufour, J. M., & Luger, R. 2017. "Identification-robust moment-based tests for Markov switching in autoregressive models." *Econometric Reviews*, 36(6-9), 713-727.

## Examples

```
set.seed(1234)
# load data used in Hamilton 1989 and extended data used in CHP 2014
y84 <- as.matrix(hamilton84GNP$GNP_logdiff)
y10 <- as.matrix(chp10GNP$GNP_logdiff)

# Set test procedure options
lmc_control = list(N = 99,
                  simdist_N = 10000,
                  getSE = TRUE)

# perform test on Hamilton 1989 data
lmc_gnp84 <- DLMMCTest(y84, p = 4, control = lmc_control)
summary(lmc_gnp84)
```

---

DLMMCTest

*Maximized Monte Carlo moment-based test for Markov switching model*

---

## Description

This function performs the maximized Monte Carlo moment-based test for Markov switching autoregressive models proposed in Dufour & Luger (2017).

## Usage

```
DLMMCTest(Y, p, control = list())
```

**Arguments**

Y	Series to be tested
p	Number of autoregressive lags.
control	List with test procedure options including: <ul style="list-style-type: none"> <li>• N: Integer determining the number of Monte Carlo simulations. Default is set to 99 as in paper.</li> <li>• simdist_N: Integer determining the number of simulations for CDF distribution approximation. Default is set to 10000.</li> <li>• getSE: Boolean indicator. If TRUE, standard errors for restricted model are estimated. If FALSE no standard errors are estimated. Default is TRUE.</li> <li>• eps: Fixed positive constant that does not depend on T used to determine lower and upper bounds on consistent set considered for nuisance parameter space.</li> <li>• CI_union: Boolean indicator determining if union between eps and confidence interval is used to determine lower and upper bound on consistent set considered for nuisance parameter space. If TRUE union is used and if FALSE only eps is used. Note that if standard errors obtained are not finite then only eps is used. Default is FALSE.</li> <li>• lambda: Numeric value for penalty on stationary constraint not being met. Default is 100.</li> <li>• stationary_ind: Boolean indicator determining if only stationary solutions should be considered if TRUE or any solution can be considered if FALSE. Default is TRUE.</li> <li>• phi_low: Vector with lower bound for autoregressive parameters when optimizing. Default is NULL.</li> <li>• phi_upp: Vector with upper bound for autoregressive parameters when optimizing. Default is NULL.</li> <li>• optim_type: String determining type of numerical optimization algorithm to use. Available options are: "pso", "GenSA", "GA". Default is "GenSA".</li> <li>• silence: Boolean indicator determining if optimization updates should be silenced if TRUE or not if FALSE. Default is FALSE.</li> <li>• threshold_stop: Numeric value determining the maximum possible p-value attainable. Default is 1.</li> <li>• type_control: List containing other optimization options specific to the numerical optimization algorithm used. This includes maximum number of iterations which is 200 by default. For other options see documentation of numerical algorithm chosen.</li> </ul>

**Value**

List of class DLMCTest (S3 object) with attributes including:

- mdl\_h0: List with restricted model attributes. This will be of class ARmdl if  $p > 0$  or Nmdl otherwise (S3 objects). See [ARmdl](#) or [Nmdl](#).
- theta\_max\_min: Value of nuisance parameters when min version of p-value is maximized as discussed in Dufour & Luger (2017) MMC procedure.

- `theta_max_prod`: Value of nuisance parameters when prod version of p-value is maximized as discussed in Dufour & Luger (2017) MMC procedure.
- `theta_low`: Lower bound on nuisance parameter values used when searching for maximum p-value.
- `theta_upp`: Upper bound on nuisance parameter values used when searching for maximum p-value.
- `S0_min`: A (1 x 4) matrix containing the four moment-based test statistics defined in (11) - (14) in Dufour & Luger (2017) when `theta_min` is used.
- `S0_prod`: A (1 x 4) matrix containing the four moment-based test statistics defined in (11) - (14) in Dufour & Luger (2017) when `theta_prod` is used.
- `F0_min`: Test statistic value for min version of Maximized Monte Carlo moment-based test.
- `F0_prod`: Test statistic value for prod version of Maximized Monte Carlo moment-based test.
- `FN_min`: A (N x 1) vector with simulated test statistics for min version of Maximized Monte Carlo moment-based test under null hypothesis.
- `FN_prod`: A (N x 1) vector with simulated test statistics for prod version of Maximized Monte Carlo moment-based test under null hypothesis.
- `pval_min`: Maximum p-value for min version of Maximized Monte Carlo moment-based test.
- `pval_prod`: Maximum p-value for prod version of Local Monte Carlo moment-based test.
- `FN_min_cv`: Vector with 90%, 95%, and 99% Monte Carlo critical values for min version of Local Monte Carlo moment-based test.
- `FN_prod_cv`: Vector with 90%, 95%, and 99% Monte Carlo critical values for prod version of Local Monte Carlo moment-based test.
- `control`: List with test procedure options used.
- `optim_min_output`: List with optimization output for min version of Maximized Monte Carlo moment-based test.
- `optim_prod_output`: List with optimization output for prod version of Maximized Monte Carlo moment-based test.

## References

Dufour, J. M., & Luger, R. 2017. "Identification-robust moment-based tests for Markov switching in autoregressive models." *Econometric Reviews*, 36(6-9), 713-727.

## Examples

```
set.seed(1234)
# load data used in Hamilton 1989 and extended data used in CHP 2014
y84 <- as.matrix(hamilton84GNP$GNP_logdiff)
y10 <- as.matrix(chp10GNP$GNP_logdiff)

# Set test procedure options
mmc_control <- list(N = 99,
                   simdist_N = 10000,
                   getSE = TRUE,
                   eps = 0.000001,
```

```

CI_union = TRUE,
lambda = 100,
stationary_ind = TRUE,
optim_type = "GenSA",
silence = FALSE,
threshold_stop = 1,
type_control = list(maxit = 200)

# perform test on Hamilton 1989 data

mmc_gnp84 <- DLMCTest(y84, p = 4, control = mmc_control)
summary(mmc_gnp84)

```

---

hamilton84GNP

*Hamilton 1984 & Hansen 1992 GNP data*


---

### Description

Hamilton 1984 & Hansen 1992 GNP data

### Usage

hamilton84GNP

### Format

This data set is used in Hansen (1992) to test the US GNP model proposed by Hamilton (1989). This series ranges from 1951Q2 to 1984Q4.

Vector of dates

**D~~GNP~~\_logdiff** US GNP log difference

**GNP** US GNP series

### Source

[https://www.ssc.wisc.edu/~bhansen/progs/jae\\_92.html](https://www.ssc.wisc.edu/~bhansen/progs/jae_92.html)

### References

Hansen, Bruce E. 1992. "The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP." *Journal of applied Econometrics* 7 (S1): S61–S82.

Hamilton, James D. 1989. "A new approach to the economic analysis of nonstationary time series and the business cycle." *Econometrica* 57 (2): 357–384.

HLRTest

*Hansen (1992) likelihood ratio test***Description**

This function performs Hansen's likelihood ratio test as described in Hansen (1992). Original source code can be found [here](#).

**Usage**

```
HLRTest(Y, p, control = list())
```

**Arguments**

- |         |  |
|---------|--|
| Y       | A (T x 1) matrix of observations.  |
| p       | Integer determining the number of autoregressive lags.   |
| control | List with test procedure options including: <ul style="list-style-type: none"> <li>• ix: List of Markov Switching parameters. 1 = just mean <math>c(1,2)</math> = mean and first param, (default: 1).</li> <li>• msvar: Boolean indicator. If TRUE, there is a switch in variance. If FALSE only switch in mean is considered. Default is FALSE.</li> <li>• qbound: Indicator that bounds q by 1-p (default: FALSE).</li> <li>• gridsize: Integer determining the number of grid points for markov switching parameters. Default is 20.</li> <li>• pgrid_from: Double determining the initial grid point for transition probabilities. Default is 0.1.</li> <li>• pgrid_by: Double determining the step size for grid points of transition probabilities. This, along with p_gridsize will determine the bounds of search space. Default is 0.075.</li> <li>• pgrid_to: Double determining the end grid point for transition probabilities. Default is 0.925.</li> <li>• mugrid_from: Double determining the minimum value of mean in second regime. Default is 0.1.</li> <li>• mugrid_by: Double determining the step size for grid points of mean in second regime. This, along with gridsize will determine the max value of mean in second regime. Default is 0.1.</li> <li>• siggrid_from: Double determining the minimum value of sigma in second regime (if msvar = TRUE). Default is 0.1.</li> <li>• siggrid_by: Double determining the step size for grid points of sigma in second regime. This, along with gridsize will determine the max value of sigma in second regime. Default is 0.1.</li> <li>• N: Integer determining the number of replications. Default is 1000.</li> <li>• nwband: Integer determining maximum bandwidth in Bartlett kernel. Critical values and p-values are returned for each bandwidth from 0: nwband as suggested in Hansen (1996). Default is 4.</li> </ul> |

- `theta_null_low`: Vector determining lower bound on parameters under the null hypothesis. Length of vector should be number of model coefficients + 1 for variance. Default is to only bound variance at  $0.01$ .
- `theta_null_upper`: Vector determining upper bound on parameters under the null hypothesis. Length of vector should be number of model coefficients + 1 for variance. Default is to no bounds (i.e. `Inf`).
- `optim_method`: String determining the type of optimization procedure used. Allowed options are "gp-optim" for general purpose optimization using `optim` from `stats` or "nl-optim" using `slsqp` from `nloptr`. Default is "gp-optim".

## Value

List of class `HLRTest` (S3 object) with model attributes including:

- `mdl_h0`: List with restricted model attributes. This will be of class `ARmdl` (S3 object). See `ARmdl`.
- `LR0`: Likelihood ratio test statistic value.
- `LRN`: A  $(N \times 1)$  vector with simulated LRT statistics under null hypothesis.
- `pval`: P-value.
- `LR_cv`: A  $(nband \times 3)$  matrix with 90%, 95%, and 99% critical values in each column respectively.
- `coef`: Vector of coefficients from restricted model and grid search that maximized standardized LRT.
- `control`: List with test procedure options used.

## References

Hansen, Bruce E. 1992. "The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP." *Journal of applied Econometrics* 7 (S1): S61–S82.

Hansen, Bruce E. 1996. "Erratum: The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP." *Journal of applied Econometrics* 7 (S1): S61–S82.

## Examples

```
# ----- Use simulated process -----
set.seed(1234)
# Define DGP of MS AR process
mdl_ms2 <- list(n      = 200,
              mu      = c(5,1),
              sigma   = c(1,1),
              phi     = c(0.5),
              k       = 2,
              P       = rbind(c(0.90, 0.10),
                              c(0.10, 0.90)))

# Simulate process using simuMSAR() function
y_ms_simu <- simuMSAR(mdl_ms2)
```

```

hlrt_control <- list(ix      = 1,
                    gridsize = 5,
                    p_gridsize = 9,
                    p_stepsize = 0.1,
                    mugrid_from = 0,
                    mugrid_by = 1)

hlrt <- HLRTTest(y_ms_simu$, p = 1, control = hlrt_control)
summary(hlrt)

```

---

HMmdl

*Hidden Markov model*


---

### Description

This function estimates a Hidden Markov model with  $k$  regimes.

### Usage

```
HMmdl(Y, k, control = list())
```

### Arguments

- |         |   |
|---------|---|
| Y       | a ( $T \times q$ ) matrix of observations.  |
| k       | integer determining the number of regimes to use in estimation. Must be greater than or equal to 2.   |
| control | List with model options including: <ul style="list-style-type: none"> <li>• <code>getSE</code>: Boolean. If TRUE standard errors are computed and returned. If FALSE standard errors are not computed. Default is TRUE.</li> <li>• <code>msmu</code>: Boolean. If TRUE model is estimated with switch in mean. If FALSE model is estimated with constant mean. Default is TRUE.</li> <li>• <code>msvar</code>: Boolean. If TRUE model is estimated with switch in variance. If FALSE model is estimated with constant variance. Default is TRUE.</li> <li>• <code>init_theta</code>: vector of initial values. vector must contain <math>(1 \times q)</math> vector <math>\mu</math>, <math>\text{vech}(\sigma)</math>, and <math>\text{vec}(P)</math> where <math>\sigma</math> is a <math>(q \times q)</math> covariance matrix. This is optional. Default is NULL, in which case <code>initVals_MSARmdl</code> is used to generate initial values.</li> <li>• <code>method</code>: string determining which method to use. Options are 'EM' for EM algorithm or 'MLE' for Maximum Likelihood Estimation.</li> <li>• <code>maxit</code>: integer determining the maximum number of EM iterations.</li> <li>• <code>thtol</code>: double determining the convergence criterion for the absolute difference in parameter estimates <math>\theta</math> between iterations. Default is <math>1e-6</math>.</li> </ul> |



- `maxit_converge`: integer determining the maximum number of initial values attempted until solution is finite. For example, if parameters in `theta` or `logLike` are NaN another set of initial values (up to `maxit_converge`) is attempted until finite values are returned. This does not occur frequently for most types of data but may be useful in some cases. Once finite values are obtained, this counts as one iteration towards `use_diff_init`. Default is 500.
- `use_diff_init`: integer determining how many different initial values to try (that do not return NaN; see `maxit_converge`). Default is 1.
- `mle_variance_constraint`: double used to determine the lower bound on the smallest eigenvalue for the covariance matrix of each regime. Default is  $1e-3$ .
- `mle_theta_low`: Vector with lower bounds on parameters (Used only if `method = "MLE"`). Default is NULL.
- `mle_theta_upper`: Vector with upper bounds on parameters (Used only if `method = "MLE"`). Default is NULL.

## Value

List of class `HMmdl` (S3 object) with model attributes including:

- `y`: a  $(T \times q)$  matrix of observations.
- `fitted`: a  $(T \times q)$  matrix of fitted values.
- `resid`: a  $(T \times q)$  matrix of residuals.
- `mu`: a  $(k \times q)$  matrix of estimated means of each process.
- `stdev`: List with  $k$   $(q \times 1)$  vector of estimated standard deviation of each process.
- `sigma`: List with  $k$   $(q \times q)$  estimated covariance matrix.
- `theta`: vector containing: `mu` and `vech(sigma)`.
- `theta_mu_ind`: vector indicating location of mean with 1 and 0 otherwise.
- `theta_sig_ind`: vector indicating location of variance and covariances with 1 and 0 otherwise.
- `theta_var_ind`: vector indicating location of variances with 1 and 0 otherwise.
- `theta_var_ind`: vector indicating location of transition matrix elements with 1 and 0 otherwise.
- `n`: number of observations (same as `T`).
- `q`: number of series.
- `k`: number of regimes in estimated model.
- `P`: a  $(k \times k)$  transition matrix.
- `pinf`: a  $(k \times 1)$  vector with limiting probabilities of each regime.
- `St`: a  $(T \times k)$  vector with smoothed probabilities of each regime at each time `t`.
- `deltath`: double with maximum absolute difference in vector `theta` between last iteration.
- `iterations`: number of EM iterations performed to achieve convergence (if less than `maxit`).
- `theta_0`: vector of initial values used.
- `init_used`: number of different initial values used to get a finite solution. See description of input `maxit_converge`.

- `msmu`: Boolean. If TRUE model was estimated with switch in mean. If FALSE model was estimated with constant mean.
- `msvar`: Boolean. If TRUE model was estimated with switch in variance. If FALSE model was estimated with constant variance.
- `control`: List with model options used.
- `logLike`: log-likelihood.
- `AIC`: Akaike information criterion.
- `BIC`: Bayesian (Schwarz) information criterion.
- `Hess`: Hessian matrix. Approximated using `hessian` and only returned if `getSE=TRUE`.
- `info_mat`: Information matrix. Computed as the inverse of `-Hess`. If matrix is not PD then nearest PD matrix is obtained using `nearest_spd`. Only returned if `getSE=TRUE`.
- `nearPD_used`: Boolean determining whether `nearPD` function was used on `info_mat` if TRUE or not if FALSE. Only returned if `getSE=TRUE`.
- `theta_se`: standard errors of parameters in theta. Only returned if `getSE=TRUE`.
- `trace`: List with Lists of estimation output for each initial value used due to `use_diff_init > 1`.

## References

- Dempster, A. P., N. M. Laird, and D. B. Rubin. 1977. "Maximum Likelihood from Incomplete Data via the EM Algorithm." *Journal of the Royal Statistical Society. Series B* 39 (1): 1–38..
- Hamilton, James D. 1990. "Analysis of time series subject to changes in regime." *Journal of econometrics*, 45 (1-2): 39–70.
- Krolzig, Hans-Martin. 1997. "The markov-switching vector autoregressive model.". Springer.

## See Also

[Nmdl](#)

---

LMCLRTest

*Monte Carlo Likelihood Ratio Test*

---

## Description

This function performs the Local Monte Carlo likelihood ratio test (LMC-LRT) proposed in Rodriguez-Rondon & Dufour (2022).

## Usage

```
LMCLRTest(Y, p, k0, k1, control = list())
```

## Arguments

Y	Series to be tested. Must be a (T x q) matrix.
p	Number of autoregressive lags. Must be greater than or equal to 0.
k0	Number of regimes under null hypothesis. Must be greater than or equal to 1.
k1	Number of regimes under alternative hypothesis. Must be greater than k0.
control	List with test procedure options including: <ul style="list-style-type: none"> <li>• N: Integer determining the number of Monte Carlo simulations. Default is set to 99 as in paper.</li> <li>• burnin: Number of simulated observations to remove from beginning. Default is 100.</li> <li>• converge_check: String of NULL determining if convergence of model(s) should be verified. Allowed inputs are: "null", "alt", "both", or NULL. If NULL (default) no model convergence is verified.</li> <li>• workers: Integer determining the number of workers to use for parallel computing version of test. Note that parallel pool must already be open. Default is 0.</li> <li>• mdl_h0_control: List with restricted model options. See <a href="#">Nmdl</a>, <a href="#">ARmdl</a>, <a href="#">VARmdl</a>, <a href="#">HMmdl</a>, <a href="#">MSARmdl</a>, or <a href="#">MSVARmdl</a> documentation for available and default values.</li> <li>• mdl_h1_control: List with unrestricted model options. See <a href="#">HMmdl</a>, <a href="#">MSARmdl</a>, or <a href="#">MSVARmdl</a> documentation for available and default values.</li> <li>• use_diff_init_sim: Value which determines the number of initial values to use when estimating models for null distribution. Default is set to use the same as specified in <code>mdl_h0_control</code> and <code>mdl_h1_control</code>.</li> </ul>

## Value

List of class LMCLRTest (S3 object) with attributes including:

- `mdl_h0`: List with restricted model attributes. See [Nmdl](#), [ARmdl](#), [VARmdl](#), [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for return values.
- `mdl_h1`: List with unrestricted model attributes. See [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for return values.
- `LRT_0`: Value of test statistic from observed data.
- `LRN`: A (N x 1) vector of test statistics from data simulated under the null hypothesis.
- `pval`: P-value of Local Monte Carlo Likelihood Ratio Test.
- `LRN_cv`: Vector with 90%, 95%, and 99% Monte Carlo critical values (from vector LRN).
- `control`: List with test procedure options used.

## References

- Rodriguez-Rondon, Gabriel and Jean-Marie Dufour. 2022. "Simulation-Based Inference for Markov Switching Models" *JSM Proceedings, Business and Economic Statistics Section: American Statistical Association*.
- Rodriguez-Rondon, Gabriel and Jean-Marie Dufour. 2023. "Monte Carlo Likelihood Ratio Tests for Markov Switching Models." *Unpublished manuscript*.

**Examples**

```

set.seed(1234)
# Define DGP of MS AR process
mdl_ms2 <- list(n      = 200,
               mu     = c(5,10),
               sigma  = c(1,4),
               phi    = c(0.5),
               k      = 2,
               P      = rbind(c(0.90, 0.10),
                              c(0.10, 0.90)))

# Simulate process using simuMSAR() function
y_ms_simu <- simuMSAR(mdl_ms2)

# ----- MS-AR example ----- #
# Set test procedure options
lmc_control = list(N = 19,
                  burnin = 100,
                  converge_check = NULL,
                  mdl_h0_control = list(const = TRUE,
                                         getSE = TRUE),
                  mdl_h1_control = list(msmu = TRUE,
                                         msvar = TRUE,
                                         getSE = TRUE,
                                         method = "EM",
                                         maxit = 300,
                                         use_diff_init = 1))

lmctest <- LMCLRTTest(y_ms_simu$y, p = 1, k0 = 1 , k1 = 2, lmc_control)
summary(lmctest)

```

---

MCpval

*Monte Carlo P-value*


---

**Description**

This function computes the Monte Carlo P-value.

**Usage**

```
MCpval(test_stat, null_vec, type = "geq")
```

**Arguments**

test_stat	Test statistic under the alternative (e.g. $S_0$ ).
null_vec	A ( $N \times 1$ ) vector with test statistic under the null hypothesis.

type String determining type of test. options are: "geq" for right-tail test, "leq" for left-tail test, "abs" for absolute value test and "two-tail" for two-tail test.

### Value

MC p-value of test

### References

Dufour, Jean-Marie 2006. "Monte Carlo tests with nuisance parameters: A general approach to finite-sample inference and nonstandard asymptotics". *Journal of Econometrics*, 133(2), 443-477.

Dufour, Jean-Marie, and Richard Luger. 2017. "Identification-robust moment-based tests for Markov switching in autoregressive models". *Econometric Reviews*, 36(6-9), 713-727.

---

MMCLRTTest

*Maximized Monte Carlo Likelihood Ratio Test*

---

### Description

This function performs the Maximized Monte Carlo likelihood ratio test (MMC-LRT) proposed in Rodriguez-Rondon & Dufour (2022).

### Usage

```
MMCLRTTest(Y, p, k0, k1, control = list())
```

### Arguments

Y	Series to be tested. Must be a (T x q) matrix.
p	Number of autoregressive lags. Must be greater than or equal to 0.
k0	Number of regimes under null hypothesis. Must be greater than or equal to 1.
k1	Number of regimes under alternative hypothesis. Must be greater than k0.
control	List with test procedure options including: <ul style="list-style-type: none"> <li>• N: Integer determining the number of Monte Carlo simulations. Default is set to 99 as in paper.</li> <li>• burnin: Number of simulated observations to remove from beginning. Default is 100.</li> <li>• converge_check: String of NULL determining if convergence of model(s) should be verified. Allowed inputs are: "null", "alt", "both", or NULL. If NULL (default) no model convergence is verified.</li> <li>• workers: Integer determining the number of workers to use for parallel computing version of test. Note that parallel pool must already be open. Default is 0.</li> <li>• type: String that determines the type of optimization algorithm used. Arguments allowed are: "pso", "GenSA", and "GA". Default is "pso".</li> </ul>

- `eps`: Double determining the constant value that defines a consistent set for search. Default is  $0.1$ .
- `CI_union`: Boolean determining if union of set determined by `eps` and confidence set should be used to define consistent set for search. Default is `TRUE`.
- `lambda`: Double determining penalty on nonlinear constraint. Default is `100`.
- `stationary_constraint`: Boolean determining if only stationary solutions are considered (if `TRUE`) or not (if `FALSE`). Default is `TRUE`.
- `phi_low`: Vector with lower bound for autoregressive parameters when optimizing. Default is `NULL`.
- `phi_upp`: Vector with upper bound for autoregressive parameters when optimizing. Default is `NULL`.
- `P_low`: Value with lower bound for transition probabilities when optimizing. Default is  $0$ .
- `P_upp`: Value with upper bound for transition probabilities when optimizing. Default is  $1$ .
- `variance_constraint`: Double used to determine the lower bound for variance in parameter set for search. Value should be between  $0$  and  $1$  as it is multiplied by consistent point estimates of variances. Default is  $0.01$  (i.e., 1% of consistent point estimates).
- `silence`: Boolean determining if optimization steps should be silenced (if `TRUE`) or not (if `FALSE`). Default is `FALSE`.
- `threshold_stop`: Double determining the global optimum of function. Default is  $1$ .
- `mdl_h0_control`: List with restricted model options. See [Nmdl](#), [ARmdl](#), [VARmdl](#), [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for available and default values.
- `mdl_h1_control`: List with unrestricted model options. See [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for available and default values.
- `use_diff_init_sim`: Value which determines the number of initial values to use when estimating models for null distribution. Default is set to use the same as specified in `mdl_h0_control` and `mdl_h1_control`.
- `type_control`: List with optimization algorithm options. See [psoptim](#), [GenSA](#), [ga](#). Default is to set `list(maxit = 200)` so that maximum number of iterations is `200`.

## Value

List of class `LMCLRTTest` (S3 object) with attributes including:

- `mdl_h0`: List with restricted model attributes. See [Nmdl](#), [ARmdl](#), [VARmdl](#), [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for return values.
- `mdl_h1`: List with unrestricted model attributes. See [HMmdl](#), [MSARmdl](#), or [MSVARmdl](#) documentation for return values.
- `LRT_0`: Value of test statistic from observed data.

- LRN: A  $(N \times 1)$  vector of test statistics from data simulated under the null hypothesis.
- pval: P-value of Local Monte Carlo Likelihood Ratio Test.
- LRN\_cv: Vector with 90%, 95%, and 99% Monte Carlo critical values (from vector LRN).
- control: List with test procedure options used.

## References

Rodriguez-Rondon, Gabriel and Jean-Marie Dufour. 2022. "Simulation-Based Inference for Markov Switching Models" *JSM Proceedings, Business and Economic Statistics Section: American Statistical Association*.

Rodriguez-Rondon, Gabriel and Jean-Marie Dufour. 2023. "Monte Carlo Likelihood Ratio Tests for Markov Switching Models." *Unpublished manuscript*.

## Examples

```
set.seed(1234)
# Define DGP of MS AR process
mdl_ms2 <- list(n      = 200,
              mu      = c(5,10),
              sigma   = c(1,4),
              phi     = c(0.5),
              k       = 2,
              P       = rbind(c(0.90, 0.10),
                              c(0.10, 0.90)))

# Simulate process using simuMSAR() function
y_ms_simu <- simuMSAR(mdl_ms2)

# Set test procedure options
mmc_control = list(N = 19,
                  burnin = 100,
                  converge_check = NULL,
                  eps = 0.1,
                  CI_union = TRUE,
                  silence = FALSE,
                  threshold_stop = 0.05 + 1e-6,
                  type = "pso",
                  maxit = 100,
                  mdl_h0_control = list(const = TRUE,
                                         getSE = TRUE),
                  mdl_h1_control = list(msmu = TRUE,
                                         msvar = TRUE,
                                         getSE = TRUE,
                                         method = "EM",
                                         maxit = 300,
                                         use_diff_init = 1))

MMCtest <- MMCLRTTest(y_ms_simu$y, p = 1 , k0 = 1 , k1 = 2, mmc_control)
summary(MMCtest)
```

MSARmdl

*Markov-switching autoregressive model***Description**

This function estimates a Markov-switching autoregressive model

**Usage**

```
MSARmdl(Y, p, k, control = list())
```

**Arguments**

- |         |  |
|---------|--|
| Y       | (T x 1) vector with observational data.  |
| p       | integer for the number of lags to use in estimation. Must be greater than or equal to 1.   |
| k       | integer for the number of regimes to use in estimation. Must be greater than or equal to 2.  |
| control | List with model options including: <ul style="list-style-type: none"> <li>• <code>getSE</code>: Boolean. If TRUE standard errors are computed and returned. If FALSE standard errors are not computed. Default is TRUE.</li> <li>• <code>msmu</code>: Boolean. If TRUE model is estimated with switch in mean. If FALSE model is estimated with constant mean. Default is TRUE.</li> <li>• <code>msvar</code>: Boolean. If TRUE model is estimated with switch in variance. If FALSE model is estimated with constant variance. Default is TRUE.</li> <li>• <code>init_theta</code>: vector of initial values. vector must contain (1 x q) vector <math>\mu</math>, <math>\text{vech}(\sigma)</math>, and <math>\text{vec}(P)</math> where <math>\sigma</math> is a (q x q) covariance matrix. This is optional. Default is NULL, in which case <code>initVals_MSARmdl</code> is used to generate initial values.</li> <li>• <code>method</code>: string determining which method to use. Options are 'EM' for EM algorithm or 'MLE' for Maximum Likelihood Estimation.</li> <li>• <code>maxit</code>: integer determining the maximum number of EM iterations.</li> <li>• <code>thtol</code>: double determining the convergence criterion for the absolute difference in parameter estimates <math>\theta</math> between iterations. Default is <math>1e-6</math>.</li> <li>• <code>maxit_converge</code>: integer determining the maximum number of initial values attempted until solution is finite. For example, if parameters in <math>\theta</math> or <code>logLike</code> are NaN another set of initial values (up to <code>maxit_converge</code>) is attempted until finite values are returned. This does not occur frequently for most types of data but may be useful in some cases. Once finite values are obtained, this counts as one iteration towards <code>use_diff_init</code>. Default is 500.</li> </ul> |



- `use_diff_init`: integer determining how many different initial values to try (that do not return NaN; see `maxit_converge`). Default is 1.
- `mle_stationary_constraint`: Boolean determining if only stationary solutions are considered (if TRUE) or not (if FALSE). Default is TRUE.
- `mle_variance_constraint`: Double used to determine the lower bound for variance in each regime. Value should be between 0 and 1 as it is multiplied by single regime variance. Default is 0.01 (i.e., 1% of single regime variance).
- `mle_theta_low`: Vector with lower bounds on parameters (Used only if `method = "MLE"`). Default is NULL.
- `mle_theta_upper`: Vector with upper bounds on parameters (Used only if `method = "MLE"`). Default is NULL.

## Value

List of class MSARmdl (S3 object) with model attributes including:

- `y`: a  $(T \times 1)$  matrix of observations.
- `X`: a  $(T-p \times p + \text{const})$  matrix of lagged observations with a leading column of 1s.
- `x`: a  $(T-p \times p)$  matrix of lagged observations.
- `fitted`: a  $(T \times 1)$  matrix of fitted values.
- `resid`: a  $(T \times 1)$  matrix of residuals.
- `inter`: a  $(k \times 1)$  vector of estimated intercepts of each process.
- `mu`: a  $(k \times 1)$  vector of estimated means of each process.
- `phi`: estimates of autoregressive coefficients.
- `stdev`: a  $(k \times 1)$  vector of estimated standard deviation of each process.
- `sigma`: a  $(k \times 1)$  estimated covariance matrix.
- `theta`: vector containing: `mu` and `vech(sigma)`.
- `theta_mu_ind`: vector indicating location of mean with 1 and 0 otherwise.
- `theta_sig_ind`: vector indicating location of variances with 1 and 0 otherwise.
- `theta_var_ind`: vector indicating location of variances with 1 and 0 otherwise. This is the same as `theta_sig_ind` in MSARmdl.
- `theta_P_ind`: vector indicating location of transition matrix elements with 1 and 0 otherwise.
- `stationary`: Boolean indicating if process is stationary if TRUE or non-stationary if FALSE.
- `n`: number of observations (same as T).
- `p`: number of autoregressive lags.
- `q`: number of series. This is always 1 in MSARmdl.
- `k`: number of regimes in estimated model.
- `P`: a  $(k \times k)$  transition matrix.
- `pinf`: a  $(k \times 1)$  vector with limiting probabilities of each regime.
- `St`: a  $(T \times k)$  vector with smoothed probabilities of each regime at each time t.

- `deltath`: double with maximum absolute difference in vector `theta` between last iteration.
- `iterations`: number of EM iterations performed to achieve convergence (if less than `maxit`).
- `theta_0`: vector of initial values used.
- `init_used`: number of different initial values used to get a finite solution. See description of input `maxit_converge`.
- `msmu`: Boolean. If TRUE model was estimated with switch in mean. If FALSE model was estimated with constant mean.
- `msvar`: Boolean. If TRUE model was estimated with switch in variance. If FALSE model was estimated with constant variance.
- `control`: List with model options used.
- `logLike`: log-likelihood.
- `AIC`: Akaike information criterion.
- `BIC`: Bayesian (Schwarz) information criterion.
- `Hess`: Hessian matrix. Approximated using `hessian` and only returned if `getSE=TRUE`.
- `info_mat`: Information matrix. Computed as the inverse of `-Hess`. If matrix is not PD then nearest PD matrix is obtained using `nearest_spd`. Only returned if `getSE=TRUE`.
- `nearPD_used`: Boolean determining whether `nearPD` function was used on `info_mat` if TRUE or not if FALSE. Only returned if `getSE=TRUE`.
- `theta_se`: standard errors of parameters in `theta`. Only returned if `getSE=TRUE`.
- `trace`: List with Lists of estimation output for each initial value used due to `use_diff_init > 1`.

## References

- Dempster, A. P., N. M. Laird, and D. B. Rubin. 1977. "Maximum Likelihood from Incomplete Data via the EM Algorithm." *Journal of the Royal Statistical Society. Series B* 39 (1): 1–38..
- Hamilton, James D. 1990. "Analysis of time series subject to changes in regime." *Journal of econometrics*, 45 (1-2): 39–70.

## See Also

[ARmdl](#)

## Examples

```
# ----- Use simulated process -----
set.seed(1234)
# Define DGP of MS AR process
mdl_ms2 <- list(n      = 200,
               mu      = c(5,10),
               sigma   = c(1,4),
               phi     = c(0.5),
               k       = 2,
               P       = rbind(c(0.90, 0.10),
                               c(0.10, 0.90)))
```

```

# Simulate process using simuMSAR() function
y_ms_simu <- simuMSAR mdl_ms2)

# Set options for model estimation
control <- list(msmu = TRUE,
               msvar = TRUE,
               method = "EM",
               use_diff_init = 1)

# Estimate model

ms_mdl <- MSARmdl(y_ms_simu$y, p = 1, k = 2, control)
summary(ms_mdl)

```

---

MSVARmdl

*Markov-switching vector autoregressive model*


---

## Description

This function estimates a Markov-switching vector autoregressive model

## Usage

```
MSVARmdl(Y, p, k, control = list())
```

## Arguments

- |         |   |
|---------|---|
| Y       | (T × q) vector with observational data.   |
| p       | integer for the number of lags to use in estimation. Must be greater than or equal to 0.  |
| k       | integer for the number of regimes to use in estimation. Must be greater than or equal to 2.   |
| control | List with optimization options including: <ul style="list-style-type: none"> <li>• <code>getSE</code>: Boolean. If TRUE standard errors are computed and returned. If FALSE standard errors are not computed. Default is TRUE.</li> <li>• <code>msmu</code>: Boolean. If TRUE model is estimated with switch in mean. If FALSE model is estimated with constant mean. Default is TRUE.</li> <li>• <code>msvar</code>: Boolean. If TRUE model is estimated with switch in variance. If FALSE model is estimated with constant variance. Default is TRUE.</li> <li>• <code>init_theta</code>: vector of initial values. vector must contain (1 × q) vector <math>\mu</math>, <math>\text{vech}(\sigma)</math>, and <math>\text{vec}(P)</math> where <math>\sigma</math> is a (q × q) covariance matrix. This is optional. Default is NULL, in which case <code>initVals_MSARmdl</code> is used to generate initial values.</li> </ul> |

- `method`: string determining which method to use. Options are 'EM' for EM algorithm or 'MLE' for Maximum Likelihood Estimation.
- `maxit`: integer determining the maximum number of EM iterations.
- `thtol`: double determining the convergence criterion for the absolute difference in parameter estimates  $\theta$  between iterations. Default is  $1e-6$ .
- `maxit_converge`: integer determining the maximum number of initial values attempted until solution is finite. For example, if parameters in  $\theta$  or `logLike` are NaN another set of initial values (up to `maxit_converge`) is attempted until finite values are returned. This does not occur frequently for most types of data but may be useful in some cases. Once finite values are obtained, this counts as one iteration towards `use_diff_init`. Default is 500.
- `use_diff_init`: integer determining how many different initial values to try (that do not return NaN; see `maxit_converge`). Default is 1.
- `mle_stationary_constraint`: Boolean determining if only stationary solutions are considered (if TRUE) or not (if FALSE). Default is TRUE.
- `mle_variance_constraint`: double used to determine the lower bound on the smallest eigenvalue for the covariance matrix of each regime. Default is  $1e-3$ .
- `mle_theta_low`: Vector with lower bounds on parameters (Used only if `method = "MLE"`). Default is NULL.
- `mle_theta_upper`: Vector with upper bounds on parameters (Used only if `method = "MLE"`). Default is NULL.

## Value

List of class MSVARmdl (S3 object) with model attributes including:

- `y`: a  $(T-p \times q)$  matrix of observations.
- `X`: a  $(T-p \times p \times q + \text{const})$  matrix of lagged observations with a leading column of 1s.
- `x`: a  $(T-p \times p \times q)$  matrix of lagged observations.
- `resid`: a  $(T-p \times q)$  matrix of residuals.
- `inter`: a  $(k \times q)$  matrix of estimated intercepts of each process.
- `mu`: a  $(k \times q)$  matrix of estimated means of each process.
- `phi`: estimates of autoregressive coefficients.
- `Fmat`: Companion matrix containing autoregressive coefficients.
- `stdev`: List with  $k$   $(q \times q)$  matrices with estimated standard deviation on the diagonal.
- `sigma`: List with  $k$   $(q \times q)$  matrices with estimated covariance matrix.
- `theta`: vector containing: `mu` and `vech(sigma)`.
- `theta_mu_ind`: vector indicating location of mean with 1 and 0 otherwise.
- `theta_sig_ind`: vector indicating location of variance and covariances with 1 and 0 otherwise.
- `theta_var_ind`: vector indicating location of variances with 1 and 0 otherwise.
- `theta_P_ind`: vector indicating location of transition matrix elements with 1 and 0 otherwise.

- stationary: Boolean indicating if process is stationary if TRUE or non-stationary if FALSE.
- n: number of observations (same as T).
- p: number of autoregressive lags.
- q: number of series.
- k: number of regimes in estimated model.
- P: a (k x k) transition matrix.
- pinf: a (k x 1) vector with limiting probabilities of each regime.
- St: a (T x k) vector with smoothed probabilities of each regime at each time t.
- deltath: double with maximum absolute difference in vector theta between last iteration.
- iterations: number of EM iterations performed to achieve convergence (if less than maxit).
- theta\_0: vector of initial values used.
- init\_used: number of different initial values used to get a finite solution. See description of input maxit\_converge.
- msmu: Boolean. If TRUE model was estimated with switch in mean. If FALSE model was estimated with constant mean.
- msvar: Boolean. If TRUE model was estimated with switch in variance. If FALSE model was estimated with constant variance.
- control: List with model options used.
- logLike: log-likelihood.
- AIC: Akaike information criterion.
- BIC: Bayesian (Schwarz) information criterion.
- Hess: Hessian matrix. Approximated using [hessian](#) and only returned if getSE=TRUE.
- info\_mat: Information matrix. Computed as the inverse of -Hess. If matrix is not PD then nearest PD matrix is obtained using [nearest\\_spd](#). Only returned if getSE=TRUE.
- nearPD\_used: Boolean determining whether nearPD function was used on info\_mat if TRUE or not if FALSE. Only returned if getSE=TRUE.
- theta\_se: standard errors of parameters in theta. Only returned if getSE=TRUE.
- trace: List with Lists of estimation output for each initial value used due to use\_diff\_init > 1.

List with model characteristics

## References

- Dempster, A. P., N. M. Laird, and D. B. Rubin. 1977. "Maximum Likelihood from Incomplete Data via the EM Algorithm." *Journal of the Royal Statistical Society. Series B* 39 (1): 1–38..
- Krolzig, Hans-Martin. 1997. "The markov-switching vector autoregressive model.". Springer.

## See Also

[VARmdl](#)

**Examples**

```

set.seed(123)
# Define DGP of MS VAR process
mdl_msvar2 <- list(n      = 200,
                  p      = 1,
                  q      = 2,
                  mu     = rbind(c(5, -2),
                                c(10, 2)),
                  sigma  = list(rbind(c(5.0, 1.5),
                                      c(1.5, 1.0)),
                                rbind(c(7.0, 3.0),
                                      c(3.0, 2.0))),
                  phi    = rbind(c(0.50, 0.30),
                                c(0.20, 0.70)),
                  k      = 2,
                  P      = rbind(c(0.90, 0.10),
                                c(0.10, 0.90)))

# Simulate process using simuMSVAR() function
y_msvar_simu <- simuMSVAR(mdl_msvar2)

# Set options for model estimation
control <- list(msmu   = TRUE,
               msvar  = TRUE,
               method = "EM",
               use_diff_init = 1)

# Estimate model

y_msvar_mdl <- MSVARmdl(y_msvar_simu$y, p = 2, k = 2, control)
summary(y_msvar_mdl)

```

---

Nmdl

*Normal distribution model*


---

**Description**

This function estimates a univariate or multivariate normally distributed model. This can be used for the null hypothesis of a linear model against an alternative hypothesis of a HMM with  $k$  regimes.

**Usage**

```
Nmdl(Y, control = list())
```

**Arguments**

**Y** a ( $T \times q$ ) matrix of observations.  
**control** List with model options including:

- `const`: Boolean determining whether to estimate model with constant if TRUE or not if FALSE. Default is TRUE.
- `getSE`: Boolean determining whether to compute standard errors of parameters if TRUE or not if FALSE. Default is TRUE.

## Value

List of class `Nmdl` (S3 object) with model attributes including:

- `y`: a  $(T \times q)$  matrix of observations.
- `fitted`: a  $(T \times q)$  matrix of fitted values.
- `resid`: a  $(T \times q)$  matrix of residuals.
- `mu`: a  $(1 \times q)$  vector of estimated means of each process.
- `stdev`: a  $(q \times 1)$  vector of estimated standard deviation of each process.
- `sigma`: a  $(q \times q)$  estimated covariance matrix.
- `theta`: vector containing: `mu` and `vech(sigma)`.
- `theta_mu_ind`: vector indicating location of mean with 1 and 0 otherwise.
- `theta_sig_ind`: vector indicating location of variance and covariances with 1 and 0 otherwise.
- `theta_var_ind`: vector indicating location of variances with 1 and 0 otherwise.
- `n`: number of observations (same as `T`).
- `q`: number of series.
- `k`: number of regimes. This is always 1 in `Nmdl`.
- `control`: List with model options used.
- `logLike`: log-likelihood.
- `AIC`: Akaike information criterion.
- `BIC`: Bayesian (Schwarz) information criterion.
- `Hess`: Hessian matrix. Approximated using `hessian` and only returned if `getSE=TRUE`.
- `info_mat`: Information matrix. Computed as the inverse of `-Hess`. If matrix is not PD then nearest PD matrix is obtained using `nearest_spd`. Only returned if `getSE=TRUE`.
- `nearPD_used`: Boolean determining whether `nearPD` function was used on `info_mat` if TRUE or not if FALSE. Only returned if `getSE=TRUE`.
- `theta_se`: standard errors of parameters in `theta`. Only returned if `getSE=TRUE`.

## Examples

```
set.seed(1234)

# ----- Univariate ----- #
# Define DGP
mdl_norm <- list(n      = 1000,
                q       = 1,
                mu      = as.matrix(5),
                sigma   = as.matrix(5.0))
```

```

# Simulate process using simuNorm() function
y_norm_simu <- simuNorm mdl_norm

# estimate parameters
y_norm_mdl <- Nmdl(y_norm_simu$y)
summary(y_norm_mdl)

# ----- Multivariate ----- #
# Define DGP
mdl_norm <- list(n      = 1000,
                q      = 2,
                mu     = c(5, -2),
                sigma  = rbind(c(5.0, 1.5),
                               c(1.5, 1.0)))

# Simulate process using simuNorm() function
y_norm_simu <- simuNorm(mdl_norm)

# estimate parameters
y_norm_mdl <- Nmdl(y_norm_simu$y)
summary(y_norm_mdl)

```

---

simuAR

*Simulate autoregressive process*


---

## Description

This function simulates an autoregressive process.

## Usage

```
simuAR(mdl_h0, burnin = 100)
```

## Arguments

mdl_h0	List containing the following DGP parameters <ul style="list-style-type: none"> <li>• n: Length of series.</li> <li>• mu: Mean of process.</li> <li>• sigma: Standard deviation of process.</li> <li>• phi: Vector of autoregressive coefficients.</li> <li>• eps: An optional (T+burnin x q) matrix with standard normal errors to be used. Errors will be generated if not provided.</li> </ul>
burnin	Number of simulated observations to remove from beginning. Default is 100.

## Value

List with simulated autoregressive series and its DGP parameters.



**Examples**

```

set.seed(1234)
# Define DGP of AR process
mdl_ar <- list(n      = 500,
              mu      = 5,
              sigma    = 2,
              phi      = c(0.5,0.2))

# Simulate process using simuAR() function
y_simu <- simuAR(mdl_ar)

plot(y_simu)

```

---

simuHMM

*Simulate Hidden Markov model with normally distributed errors*


---

**Description**

This function simulates a Hidden Markov Model process.

**Usage**

```
simuHMM(mdl_h0, burnin = 100)
```

**Arguments**

mdl_h0	List containing the following DGP parameters <ul style="list-style-type: none"> <li>• n: Length of series.</li> <li>• k: Number of regimes.</li> <li>• mu: A (k x q) vector of means.</li> <li>• sigma: A (q x q) covariance matrix.</li> <li>• q: Number of series.</li> <li>• P: A (k x k) transition matrix (columns must sum to one).</li> <li>• eps: An optional (T+burnin x q) matrix with standard normal errors to be used. Errors will be generated if not provided.</li> </ul>
burnin	Number of simulated observations to remove from beginning. Default is 100.

**Value**

List with simulated series and its DGP parameters.

**Examples**

```

set.seed(1234)

# ----- Univariate ----- #
# Define DGP
mdl_hmm <- list(n      = 1000,
               q      = 1,
               mu     = as.matrix(c(5,
                                   -2)),
               sigma  = list(as.matrix(5.0),
                             as.matrix(7.0)),
               k      = 2,
               P      = rbind(c(0.90, 0.10),
                              c(0.10, 0.90)))

# Simulate process using simuHMM() function
y_hmm_simu <- simuHMM(mdl_hmm)

plot(y_hmm_simu)

# ----- Multivariate ----- #
# Define DGP
mdl_hmm <- list(n      = 1000,
               q      = 2,
               mu     = rbind(c(5, -2),
                              c(10, 2)),
               sigma  = list(rbind(c(5.0, 1.5),
                                   c(1.5, 1.0)),
                             rbind(c(7.0, 3.0),
                                   c(3.0, 2.0))),
               k      = 2,
               P      = rbind(c(0.90, 0.10),
                              c(0.10, 0.90)))

# Simulate process using simuHMM() function
y_hmm_simu <- simuHMM(mdl_hmm)

plot(y_hmm_simu)

```

---

simuMSAR

*Simulate Markov-switching autoregressive process*


---

**Description**

This function simulates a Markov-switching autoregressive process.

**Usage**

```
simuMSAR(mdl_h0, burnin = 100)
```

**Arguments**

mdl_h0	List containing the following DGP parameters <ul style="list-style-type: none"> <li>• n: Length of series.</li> <li>• k: Number of regimes.</li> <li>• mu: A (k x 1) vector with mean of process in each regime.</li> <li>• sigma: A (k x 1) vector with standard deviation of process in each regime.</li> <li>• phi: Vector of autoregressive coefficients.</li> <li>• P: A (k x k) transition matrix (columns must sum to one).</li> <li>• eps: An optional (T+burnin x q) matrix with standard normal errors to be used. Errors will be generated if not provided.</li> </ul>
burnin	Number of simulated observations to remove from beginning. Default is 100.

**Value**

List with simulated Markov-switching autoregressive process and its DGP properties.

**Examples**

```
set.seed(1234)
# Define DGP of MS AR process
mdl_ms2 <- list(n      = 500,
               mu      = c(5,10),
               sigma   = c(1,2),
               phi     = c(0.5, 0.2),
               k       = 2,
               P       = rbind(c(0.90, 0.10),
                               c(0.10, 0.90)))

# Simulate process using simuMSAR() function
y_ms_simu <- simuMSAR(mdl_ms2)

plot(y_ms_simu)
```

---

simuMSVAR

---

*Simulate Markov-switching vector autoregressive process*


---

**Description**

This function simulates a Markov-switching vector autoregressive process.

**Usage**

```
simuMSVAR(mdl_h0, burnin = 100)
```

**Arguments**

mdl_h0	<p>List containing the following DGP parameters</p> <ul style="list-style-type: none"> <li>• n: Length of series.</li> <li>• k: Number of regimes.</li> <li>• mu: A (k x q) matrix of means.</li> <li>• sigma: List with k (q x q) covariance matrices.</li> <li>• phi: A (q x qp) matrix of autoregressive coefficients.</li> <li>• p: Number of autoregressive lags.</li> <li>• q: Number of series.</li> <li>• P: A (k x k) transition matrix (columns must sum to one).</li> <li>• eps: An optional (T+burnin x q) matrix with standard normal errors to be used. Errors will be generated if not provided.</li> </ul>
burnin	Number of simulated observations to remove from beginning. Default is 100.

**Value**

List with simulated vector autoregressive series and its DGP parameters.

**Examples**

```

set.seed(1234)
# Define DGP of MS VAR process
mdl_msvar2 <- list(n      = 1000,
                  p      = 1,
                  q      = 2,
                  mu     = rbind(c(5, -2),
                                c(10, 2)),
                  sigma  = list(rbind(c(5.0, 1.5),
                                      c(1.5, 1.0)),
                                rbind(c(7.0, 3.0),
                                      c(3.0, 2.0))),
                  phi    = rbind(c(0.50, 0.30),
                                c(0.20, 0.70)),
                  k      = 2,
                  P      = rbind(c(0.90, 0.10),
                                c(0.10, 0.90)))

# Simulate process using simuMSVAR() function
y_msvar_simu <- simuMSVAR(mdl_msvar2)

plot(y_msvar_simu)

```

---

simuNorm	<i>Simulate normally distributed process</i>
----------	--

---

## Description

This function simulates a normally distributed process.

## Usage

```
simuNorm mdl_h0, burnin = 0)
```

## Arguments

mdl_h0	List containing the following DGP parameters <ul style="list-style-type: none"><li>• n: Length of series.</li><li>• mu: A (q x 1) vector of means.</li><li>• sigma: A (q x q) covariance matrix.</li><li>• q: Number of series.</li><li>• eps: An optional (T+burnin x q) matrix with standard normal errors to be used. Errors will be generated if not provided.</li></ul>
burnin	Number of simulated observations to remove from beginning. Default is 100.

## Value

List with simulated series and its DGP parameters.

## Examples

```
set.seed(1234)
# Define DGP
mdl_norm <- list(n      = 1000,
                q       = 2,
                mu      = c(5, -2),
                sigma   = rbind(c(5.0, 1.5),
                               c(1.5, 1.0)))

# Simulate process using simuNorm() function
y_norm_simu <- simuNorm(mdl_norm)

plot(y_norm_simu)
```

---

simuVAR	<i>Simulate VAR process</i>
---------	-----------------------------

---

**Description**

This function simulates a vector autoregressive process.

**Usage**

```
simuVAR mdl_h0, burnin = 100)
```

**Arguments**

mdl_h0	List containing the following DGP parameters <ul style="list-style-type: none"> <li>• n: Length of series.</li> <li>• mu: A (q x 1) vector of means.</li> <li>• sigma: A (q x q) covariance matrix.</li> <li>• phi: A (q x qp) matrix of autoregressive coefficients.</li> <li>• p: Number of autoregressive lags.</li> <li>• q: Number of series.</li> <li>• eps: An optional (T+burnin x q) matrix with standard normal errors to be used. Errors will be generated if not provided.</li> </ul>
burnin	Number of simulated observations to remove from beginning. Default is 100.

**Value**

List with simulated vector autoregressive series and its DGP parameters.

---

USGNP	<i>US GNP data 1947Q2 - 2023Q4</i>
-------	------------------------------------

---

**Description**

US GNP data 1947Q2 - 2023Q4

**Usage**

```
USGNP
```

**Format**

This data is used in Rodriguez-Rondon & Dufour (2024). The series ranges from 1947Q2 to 2023Q4.

Vector of dates

**DAGNP\_logdiff** log difference of US GDP series

**GNP** US GNP series

**Source**

<https://fred.stlouisfed.org/graph/?g=UKDQ>

**References**

Rodriguez-Rondon, Gabriel and Jean-Marie Dufour. 2024. "Monte Carlo Likelihood Ratio Tests for Markov Switching Models." *Unpublished manuscript*.

---

VARmdl	<i>Vector autoregressive model</i>
--------	------------------------------------

---

**Description**

This function estimates a vector autoregressive model with  $p$  lags. This can be used for the null hypothesis of a linear model against an alternative hypothesis of a Markov switching vector autoregressive model with  $k$  regimes.

**Usage**

```
VARmdl(Y, p, control = list())
```

**Arguments**

Y	a ( $T \times q$ ) matrix of observations.
p	integer determining the number of autoregressive lags.
control	List with model options including: <ul style="list-style-type: none"> <li>• const: Boolean determining whether to estimate model with constant if TRUE or not if FALSE. Default is TRUE.</li> <li>• getSE: Boolean determining whether to compute standard errors of parameters if TRUE or not if FALSE. Default is TRUE.</li> </ul>

**Value**

List of class VARmdl (S3 object) with model attributes including:

- y: a ( $T-p \times q$ ) matrix of observations.
- X: a ( $T-p \times p \times q + \text{const}$ ) matrix of lagged observations with a leading column of 1s if `const=TRUE` or not if `const=FALSE`.
- x: a ( $T-p \times p \times q$ ) matrix of lagged observations.
- fitted: a ( $T-p \times q$ ) matrix of fitted values.
- resid: a ( $T-p \times q$ ) matrix of residuals.
- inter: a ( $1 \times q$ ) vector of estimated intercepts of each process.
- mu: a ( $1 \times q$ ) vector of estimated means of each process.

- `coef`: coefficient estimates. First row are the intercept (i.e., not  $\mu$ ) if `const=TRUE`. This is the same as `t(phi)` if `const=FALSE`.
- `intercept`: estimate of intercepts.
- `phi`: a  $(q \times p \times q)$  matrix of estimated autoregressive coefficients.
- `Fmat`: Companion matrix containing autoregressive coefficients.
- `stdev`: a  $(q \times 1)$  vector of estimated standard deviation of each process.
- `sigma`: a  $(q \times q)$  estimated covariance matrix.
- `theta`: vector containing:  $\mu$ , `vech(sigma)`, and `vec(t(phi))`.
- `theta_mu_ind`: vector indicating location of mean with 1 and 0 otherwise.
- `theta_sig_ind`: vector indicating location of variance and covariances with 1 and 0 otherwise.
- `theta_var_ind`: vector indicating location of variances with 1 and 0 otherwise.
- `theta_phi_ind`: vector indicating location of autoregressive coefficients with 1 and 0 otherwise.
- `stationary`: Boolean indicating if process is stationary if TRUE or non-stationary if FALSE.
- `n`: number of observations after lag transformation (i.e.,  $n = T-p$ ).
- `p`: number of autoregressive lags.
- `q`: number of series.
- `k`: number of regimes. This is always 1 in VARmdl.
- `Fmat`: matrix from companion form. Used to determine is process is stationary.
- `control`: List with model options used.
- `logLike`: log-likelihood.
- `AIC`: Akaike information criterion.
- `BIC`: Bayesian (Schwarz) information criterion.
- `Hess`: Hessian matrix. Approximated using `hessian` and only returned if `getSE=TRUE`.
- `info_mat`: Information matrix. Computed as the inverse of `-Hess`. If matrix is not PD then nearest PD matrix is obtained using `nearest_spd`. Only returned if `getSE=TRUE`.
- `nearPD_used`: Boolean determining whether `nearPD` function was used on `info_mat` if TRUE or not if FALSE. Only returned if `getSE=TRUE`.
- `theta_se`: standard errors of parameters in `theta`. Only returned if `getSE=TRUE`.

### See Also

[MSVARmdl](#)

### Examples

```
# ----- Bivariate VAR(1) process ----- #
set.seed(1234)
# Define DGP of VAR process
mdl_var <- list(n      = 1000,
               p       = 1,
               q       = 2,
               mu      = c(5, -2),
```



```
sigma = rbind(c(5.0, 1.5),
              c(1.5, 1.0)),
phi    = rbind(c(0.50, 0.30),
              c(0.20, 0.70)))

# Simulate process using simuVAR() function
y_simu <- simuVAR mdl_var

# Set options for model estimation
control <- list(const = TRUE,
               getSE = TRUE)

# Estimate model
y_var_mdl <- VARmdl(y_simu$y, p = 2, control)
summary(y_var_mdl)
```

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