

# Package ‘bspline’

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**Type** Package

**Title** B-Spline Interpolation and Regression

**Version** 2.2

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**Description** Build and use B-splines for interpolation and regression.

In case of regression, equality constraints as well as monotonicity and/or positivity of B-spline weights can be imposed. Moreover, knot positions (not only spline weights) can be part of optimized parameters too. For this end, 'bspline' is able to calculate Jacobian of basis vectors as function of knot positions. User is provided with functions calculating spline values at arbitrary points. These functions can be differentiated and integrated to obtain B-splines calculating derivatives/integrals at any point. B-splines of this package can simultaneously operate on a series of curves sharing the same set of knots. 'bspline' is written with concern about computing performance that's why the basis and Jacobian calculation is implemented in C++. The rest is implemented in R but without notable impact on computing speed.

**URL** <https://github.com/MathsCell/bspline>

**BugReports** <https://github.com/MathsCell/bspline/issues>

**License** GPL-2

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**Imports** Rcpp (>= 1.0.7), nlsic (>= 1.0.2), arrApply

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bcurve	<i>nD B-curve governed by (x,y,...) control points.</i>
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### Description

nD B-curve governed by (x,y,...) control points.

### Usage

```
bcurve(xy, n = 3)
```

### Arguments

xy	Real matrix of (x,y,...) coordinates, one control point per row.
n	Integer scalar, polynomial order of B-spline (3 by default)

### Details

The curve will pass by the first and the last points in 'xy'. The tangents at the first and last points will coincide with the first and last segments of control points. Example of signature is inspired from this [blog](#).

### Value

Function of one argument calculating B-curve. The argument is supposed to be in [0, 1] interval.

**Examples**

```

# simulate doctor's signature ;)
set.seed(71);
xy=matrix(rnorm(16), ncol=2)
tp=seq(0,1,len=301)
doc_signtr=bcurve(xy)
plot(doc_signtr(tp), t="l", xaxt='n', yaxt='n', ann=FALSE, frame.plot=FALSE,
      xlim=range(xy[,1]), ylim=range(xy[,2]))
# see where control points are
text(xy, labels=seq(nrow(xy)), col=rgb(0, 0, 0, 0.25))
# join them by segments
lines(bcurve(xy, n=1)(tp), col=rgb(0, 0, 1, 0.25))

# randomly curved wire in 3D space
## Not run:
if (requireNamespace("rgl", quietly=TRUE)) {
  xyz=matrix(rnorm(24),ncol=3)
  tp=seq(0,1,len=201)
  curv3d=bcurve(xyz)
  rgl::plot3d(curv3d(tp), t="l", decorate=FALSE)
}
## End(Not run)

```

---

bsc	<i>Basis matrix and knot Jacobian for B-spline of order 0 (step function) and higher</i>
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**Description**

This function is analogous but not equivalent to `splines:bs()` and `splines2::bSpline()`. It is also several times faster.

**Usage**

```
bsc(x, xk, n = 3L, cjac = FALSE)
```

**Arguments**

x	Numeric vector, abscissa points
xk	Numeric vector, knots
n	Integer scalar, polynomial order (3 by default)
cjac	Logical scalar, if TRUE makes to calculate Jacobian of basis vectors as function of knot positions (FALSE by default)

**Details**

For  $n=0$ , step function is defined as constant on each interval  $[x_k[i]; x_k[i+1]]$ , i.e. closed on the left and open on the right except for the last interval which is closed on the right too. The Jacobian for step function is considered 0 in every  $x$  point even if in points where  $x=x_k$ , the derivative is not defined.

For  $n=1$ , Jacobian is discontinuous in such points so for these points we take the derivative from the right.

**Value**

Numeric matrix (for `cjac=FALSE`), each column correspond to a B-spline calculated on  $x$ ; or List (for `cjac=TRUE`) with components

**mat** basis matrix of dimension  $n_x \times n_w$ , where  $n_x$  is the length of  $x$  and  $n_w=n_k-n-1$  is the number of basis vectors

**jac** array of dimension  $n_x \times (n+2) \times n_w$  where  $n+2$  is the number of support knots for each basis vector

**See Also**

`splines::bs()`, `splines2::bSpline()`

**Examples**

```
x=seq(0, 5, length.out=101)
# cubic basis matrix
n=3
m=bsc(x, xk=c(rep(0, n+1), 1:4, rep(5, n+1)), n=n)
matplot(x, m, t="l")
stopifnot(all.equal(numeric(c(m), c(splines::bs(x, knots = 1:4, degree = n, intercept = TRUE))))))
```

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 bsp

---

*Calculate B-spline values from their coefficients qw and knots xk*


---

**Description**

Calculate B-spline values from their coefficients  $qw$  and knots  $xk$

**Usage**

```
bsp(x, xk, qw, n = 3L)
```

**Arguments**

x	Numeric vector, abscissa points at which B-splines should be calculated. They are supposed to be non decreasing.
xk	Numeric vector, knots of the B-splines. They are supposed to be non decreasing.
qw	Numeric vector or matrix, coefficients of B-splines. $NROW(qw)$ must be equal to $length(xk)-n-1$ where $n$ is the next parameter
n	Integer scalar, polynomial order of B-splines, by default cubic splines are calculated.

**Details**

This function does nothing else than calculate a dot-product between a B-spline basis matrix calculated by `bsc()` and coefficients `qw`. If `qw` is a matrix, each column corresponds to a separate set of coefficients. For `x` values falling outside of `xk` range, the B-splines values are set to 0. To get a function calculating spline values at arbitrary points from `xk` and `qw`, cf. `par2bsp()`.

**Value**

Numeric matrix (column number depends on `qw` dimensions), B-spline values on `x`.

**See Also**

`bsc`, `par2bsp`

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bspline

*bspline: build and use B-splines for interpolation and regression.*

---

**Description**

Build and use B-splines for interpolation and regression. In case of regression, equality constraints as well as monotonicity requirement can be imposed. Moreover, knot positions (not only spline coefficients) can be part of optimized parameters too. User is provided with functions calculating spline values at arbitrary points. This functions can be differentiated to obtain B-splines calculating derivatives at any point. B-splines of this package can simultaneously operate on a series of curves sharing the same set of knots. 'bspline' is written with concern about computing performance that's why the basis calculation is implemented in C++. The rest is implemented in R but without notable impact on computing speed.

**bspline functions**

- `bsc`: basis matrix (implemented in C++)
- `bsp`: values of B-spline from its coefficients
- `dbsp`: derivative of B-spline
- `par2bsp`: build B-spline function from parameters
- `bspvar`: retrieve B-spline parameters from its function

- `smbbsp`: build smoothing B-spline
- `fitsmbbsp`: build smoothing B-spline with optimized knot positions
- `diffn`: finite differences

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`bsppar` *Retrieve parameters of B-splines*

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### Description

Retrieve parameters of B-splines

### Usage

`bsppar(f)`

### Arguments

`f` Function, B-splines such that returned by `par3bsp()`, `smbbsp()`, ...

### Value

List having components: `n` - polynomial order, `qw` - coefficients, `xk` - knots

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`dbsp` *Derivative of B-spline*

---

### Description

Derivative of B-spline

### Usage

`dbsp(f, nderiv = 1L, same_xk = FALSE)`

### Arguments

`f` Function, B-spline such as returned by `smbbsp()` or `par2bsp()`

`nderiv` Integer scalar  $\geq 0$ , order of derivative to calculate (1 by default)

`same_xk` Logical scalar, if TRUE, indicates to calculate derivative on the same knot grid as original function. In this case, coefficient number will be incremented by 2. Otherwise, extreme knots are removed on each side of the grid and coefficient number is maintained (FALSE by default).

### Value

Function calculating requested derivative

**Examples**

```
x=seq(0., 1., length.out=11L)
y=sin(2*pi*x)
f=smbbsp(x, y, nki=2L)
d_f=dbbsp(f)
xf=seq(0., 1., length.out=101) # fine grid for plotting
plot(xf, d_f(xf)) # derivative estimated by B-splines
lines(xf, 2.*pi*cos(2*pi*xf), col="blue") # true derivative
xk=bsppar(d_f)$xk
points(xk, d_f(xk), pch="x", col="red") # knot positions
```

---

diffn	<i>Finite differences</i>
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**Description**

Calculate  $dy/dx$  where  $x,y$  are first and the rest of columns in the entry matrix 'm'

**Usage**

```
diffn(m, ndiff = 1L)
```

**Arguments**

m	2- or more-column numeric matrix
ndiff	Integer scalar, order of finite difference (1 by default)

**Value**

Numeric matrix, first column is midpoints of  $x$ , the second and following are  $dy/dx$

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dmat	<i>Differentiation matrix</i>
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**Description**

Calculate matrix for obtaining coefficients of first-derivative B-spline. They can be calculated as  $dqw=Md \%*\% qw$ . Here,  $dqw$  are coefficients of the first derivative,  $Md$  is the matrix returned by this function, and  $qw$  are the coefficients of differentiated B-spline.

**Usage**

```
dmat(nqw = NULL, xk = NULL, n = NULL, f = NULL, same_xk = FALSE)
```

**Arguments**

nqw	Integer scalar, row number of qw matrix (i.e. degree of freedom of a B-spline)
xk	Numeric vector, knot positions
n	Integer scalar, B-spline polynomial order
f	Function from which previous parameters can be retrieved. If both f and any of previous parameters are given then explicitly set parameters take precedence over those retrieved from f.
same_xk	Logical scalar, the same meaning as in <a href="#">dbsp</a>

**Value**

Numeric matrix of size  $nqw-1 \times nqw$

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ibsp	<i>Indefinite integral of B-spline</i>
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**Description**

Indefinite integral of B-spline

**Usage**

```
ibsp(f, const = 0, nint = 1L)
```

**Arguments**

f	Function, B-spline such as returned by <code>smbsp()</code> or <code>par2bsp()</code>
const	Numeric scalar or vector of length <code>ncol(qw)</code> where <code>qw</code> is weight matrix of <code>f</code> . Defines starting value of weights for indefinite integral (0 by default).
nint	Integer scalar $\geq 0$ , defines how many times to take integral (1 by default)

**Details**

If `f` is B-spline, then following identity is held: `Dbps(ibsp(f))` is identical to `f`. Generally, it does not work in the other sens: `ibsp(Dbps(f))` is not `f` but not very far. If we can get an appropriate constant  $C=f(\min(x))$  then we can assert that `ibsp(Dbps(f), const=C)` is the same as `f`.

**Value**

Function calculating requested integral



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iknots *Estimate internal knot positions equalizing jumps in n-th derivative*

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### Description

Normalized total variation of n-th finite differences is calculated for each column in y then averaged. These averaged values are fitted by a linear spline to find knot positions that equalize the jumps of n-th derivative.

NB. This function is used internally in (fit)smbssp() and a priori has no interest to be called directly by user.

### Usage

```
iknots(x, y, nki = 1L, n = 3L)
```

### Arguments

x	Numeric vector
y	Numeric vector or matrix
nki	Integer scalar, number of internal knots to estimate (1 by default)
n	Integer scalar, polynomial order of B-spline (3 by default)

### Value

Numeric vector, estimated knot positions

---

ipk *Intervals of points in knot intervals*

---

### Description

Find first and last+1 indexes iip s.t. x[iip] belongs to interval starting at xk[iik]

### Usage

```
ipk(x, xk)
```

### Arguments

x	Numeric vector, abscissa points (must be non decreasing)
xk	Numeric vector, knots (must be non decreasing)

### Value

Integer matrix of size (2 x length(xk)-1). Indexes are 0-based

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jacw	<i>Knot Jacobian of B-spline with weights</i>
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---

**Description**

Knot Jacobian of B-spline with weights

**Usage**

```
jacw(jac, qws)
```

**Arguments**

jac	Numeric array, such as returned by <code>bsc(..., cjac=TRUE)</code>
qws	Numeric matrix, each column is a set of weights forming a B-spline. If qws is a vector, it is coerced to 1-column matrix.

**Value**

Numeric array of size  $n_x \times n_{col}(q_w) \times n_k$ , where  $n_x = \dim(jac)[1]$  and  $n_k$  is the number of knots  $\dim(jac)[3] + n + 1$  (n being polynomial order).

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par2bsp	<i>Convert parameters to B-spline function</i>
---------	--

---

**Description**

Convert parameters to B-spline function

**Usage**

```
par2bsp(n, qw, xk, covqw = NULL, sdy = NULL, sdqw = NULL)
```

**Arguments**

n	Integer scalar, polynomial order of B-splines
qw	Numeric vector or matrix, coefficients of B-splines, one set per column in case of matrix
xk	Numeric vector, knots
covqw	Numeric Matrix, covariance matrix of qw (can be estimated in <a href="#">smbasp</a> ).
sdy	Numeric vector, SD of each y column (can be estimated in <a href="#">smbasp</a> ).
sdqw	Numeric Matrix, SD of qw thus having the same dimension as qw (can be estimated in <a href="#">smbasp</a> ).

**Value**

Function, calculating B-splines at arbitrary points and having interface  $f(x, \text{select})$  where  $x$  is a vector of abscissa points. Parameter  $\text{select}$  is passed to  $qw[, \text{select}, \text{drop}=\text{FALSE}]$  and can be missing. This function will return a matrix of size  $\text{length}(x) \times \text{ncol}(qw)$  if  $\text{select}$  is missing. Elsewhere, a number of column will depend on  $\text{select}$  parameter. Column names in the result matrix will be inherited from  $qw$ .

---

parr	<i>Polynomial formulation of B-spline</i>
------	---

---

**Description**

Polynomial formulation of B-spline

**Usage**

```
parr(xk, n = 3L)
```

**Arguments**

xk	Numeric vector, knots
n	Integer scalar, polynomial order (3 by default)

**Value**

Numeric 3D array, the first index runs through  $n+1$  polynomial coefficients; the second – through  $n+1$  supporting intervals; and the last one through  $n_k - n - 1$  B-splines (here  $n_k = \text{length}(xk)$ ). Knot interval of length 0 will have corresponding coefficients set to 0.

---

smbbsp	<i>Smoothing B-spline of order <math>n \geq 0</math></i>
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---

**Description**

Optimize smoothing B-spline coefficients (smbbsp) and knot positions (fitsmbbsp) such that residual squared sum is minimized for all  $y$  columns.

**Usage**

```
smbsp(
  x,
  y,
  n = 3L,
  xki = NULL,
  nki = 1L,
  lieq = NULL,
  monotone = 0,
  positive = 0,
  mat = NULL,
  estSD = FALSE,
  tol = 1e-10
)
```

```
fitsmbsp(
  x,
  y,
  n = 3L,
  xki = NULL,
  nki = 1L,
  lieq = NULL,
  monotone = 0,
  positive = 0,
  control = list(),
  estSD = FALSE,
  tol = 1e-10
)
```

**Arguments**

x	Numeric vector, abscissa points
y	Numeric vector or matrix or data.frame, ordinate values to be smoothed (one set per column in case of matrix or data.frame)
n	Integer scalar, polynomial order of B-splines (3 by default)
xki	Numeric vector, strictly internal B-spline knots, i.e. lying strictly inside of x bounds. If NULL (by default), they are estimated with the help of <code>iknots()</code> . This vector is used as initial approximation during optimization process. Must be non decreasing if not NULL.
nki	Integer scalar, internal knot number (1 by default). When <code>nki==0</code> , it corresponds to polynomial regression. If <code>xki</code> is not NULL, this parameter is ignored.
lieq	List, equality constraints to respect by the smoothing spline, one list item per y column. By default (NULL), no constraint is imposed. Constraints are given as a 2-column matrix ( <code>xe</code> , <code>ye</code> ) where for each <code>xe</code> , an <code>ye</code> value is imposed. If a list item is NULL, no constraint is imposed on corresponding y column.
monotone	Numeric scalar or vector, if <code>monotone &gt; 0</code> , resulting B-spline weights must be increasing; if <code>monotone &lt; 0</code> , B-spline weights must be decreasing; if <code>monotone</code>

	<code>== 0</code> (default), no constraint on monotonicity is imposed. If 'monotone' is a vector it must be of length <code>ncol(y)</code> , in which case each component indicates the constraint for corresponding column of <code>y</code> .
<code>positive</code>	Numeric scalar, if <code>positive &gt; 0</code> , resulting B-spline weights must be $\geq 0$ ; if <code>positive &lt; 0</code> , B-spline weights must be decreasing; if <code>positive == 0</code> (default), no constraint on positivity is imposed. If 'positive' is a vector it must be of length <code>ncol(y)</code> , in which case each component indicates the constraint for corresponding column of <code>y</code> .
<code>mat</code>	Numeric matrix of basis vectors, if <code>NULL</code> it is recalculated by <code>bsc()</code> . If provided, it is the responsibility of the user to ensure that this matrix be adequate to <code>xki</code> vector.
<code>estSD</code>	Logical scalar, if <code>TRUE</code> , indicates to calculate: SD of each <code>y</code> column, covariance matrix and SD of spline coefficients. All these values can be retrieved with <code>bsppar()</code> call ( <code>FALSE</code> by default). These estimations are made under assumption that all <code>y</code> points have uncorrelated noise. Optional constraints are not taken into account of SD.
<code>tol</code>	Numerical scalar, relative tolerance for small singular values that should be considered as 0 if <code>s[i] &lt;= tol*s[1]</code> . This parameter is ignored if <code>estSD=FALSE</code> ( <code>1.e-10</code> by default).
<code>control</code>	List, passed through to <code>nlsic()</code> call

## Details

If constraints are set, we use `nlsic::lsie_ln()` to solve a least squares problem with equality constraints in least norm sens for each `y` column. Otherwise, `nlsic::ls_ln_svd()` is used for the whole `y` matrix. The solution of least squares problem is a vector of B-splines coefficients `qw`, one vector per `y` column. These vectors are used to define B-spline function which is returned as the result.

NB. When `nki >= length(x)-n-1` (be it from direct setting or calculated from `length(xki)`), it corresponds to spline interpolation, i.e. the resulting spline will pass exactly by `(x,y)` points (well, up to numerical precision).

Border and external knots are fixed, only strictly internal knots can move during optimization. The optimization process is constrained to respect a minimal distance between knots as well as to bound them to `x` range. This is done to avoid knots getting unsorted during iterations and/or going outside of a meaningful range.

## Value

Function, smoothing B-splines respecting optional constraints (generated by `par2bsp()`).

## See Also

`bsppar` for retrieving parameters of B-spline functions; `par2bsp` for generating B-spline function; `iknots` for estimation of knot positions

**Examples**

```
x=seq(0, 1, length.out=11)
y=sin(pi*x)+rnorm(x, sd=0.1)
# constraint B-spline to be 0 at the interval ends
fsm=smbbsp(x, y, nki=1, lieq=list(rbind(c(0, 0), c(1, 0))))
# check parameters of found B-splines
bsppar(fsm)
plot(x, y) # original "measurements"
# fine grained x
xfine=seq(0, 1, length.out=101)
lines(xfine, fsm(xfine)) # fitted B-splines
lines(xfine, sin(pi*xfine), col="blue") # original function
# visualize knot positions
xk=bsppar(fsm)$xk
points(xk, fsm(xk), pch="x", col="red")
# fit broken line with linear B-splines
x1=seq(0, 1, length.out=11)
x2=seq(1, 3, length.out=21)
x3=seq(3, 4, length.out=11)
y1=x1+rnorm(x1, sd=0.1)
y2=-2+3*x2+rnorm(x2, sd=0.1)
y3=4+x3+rnorm(x3, sd=0.1)
x=c(x1, x2, x3)
y=c(y1, y2, y3)
plot(x, y)
f=fitsmbbsp(x, y, n=1, nki=2)
lines(x, f(x))
```

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