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# Chapter 1

## Classes

### 1.1 quad – Imaginary Quadratic Field

- Classes
  - **ReducedQuadraticForm**
  - **ClassGroup**
- Functions
  - **class\_formula**
  - **class\_number**
  - **class\_group**
  - **class\_number\_bsgs**
  - **class\_group\_bsgs**

#### 1.1.1 ReducedQuadraticForm – Reduced Quadratic Form Class

##### Initialize (Constructor)

**ReducedQuadraticForm(f: list, unit: list) → *ReducedQuadraticForm***

Create ReducedQuadraticForm object.

f, unit must be list of 3 integers [a, b, c], representing a quadratic form  $ax^2 + bxy + cy^2$ . unit represents the unit form.

##### Operations

operator	explanation
$M * N$	Return the composition form of $M$ and $N$ .
$M ** a$	Return the $a$ -th powering of $M$ .
$M / N$	Division of form.
$M == N$	Return whether $M$ and $N$ are equal or not.
$M != N$	Return whether $M$ and $N$ are unequal or not.

## Methods

### 1.1.1.1 inverse

`inverse(self)` → *ReducedQuadraticForm*

Return the inverse of `self`.

### 1.1.1.2 disc

`disc(self)` → *ReducedQuadraticForm*

Return the discriminant of `self`.

## 1.1.2 ClassGroup – Class Group Class

### Initialize (Constructor)

`ClassGroup(disc: integer, cl: integer, element: integer=None)`  
→ *ClassGroup*

Create ClassGroup object.

## Methods

### 1.1.3 class\_formula

**class\_formula**(d: *integer*, uprbd: *integer*) → *integer*

Return the approximation of class number  $h$  with discriminant  $d$  using class formula.

$$\text{class formula } h = \frac{\sqrt{|d|}}{\pi} \prod_p \left( 1 - \left( \frac{d}{p} \right) \frac{1}{p} \right)^{-1}.$$

Input number  $d$  must be int, long or **Integer**.

### 1.1.4 class\_number

**class\_number**(d: *integer*, limit\_of\_d: *integer*=1000000000) → *integer*

Return the class number with the discriminant  $d$  by counting reduced forms.

$d$  is not only fundamental discriminant.

Input number  $d$  must be int, long or **Integer**.

### 1.1.5 class\_group

**class\_group**(d: *integer*, limit\_of\_d: *integer*=1000000000) → *integer*

Return the class number and the class group with the discriminant  $d$  by counting reduced forms.

$d$  is not only fundamental discriminant.

Input number  $d$  must be int, long or **Integer**.

### 1.1.6 class\_number\_bsgs

**class\_number\_bsgs**(d: *integer*) → *integer*

Return the class number with the discriminant  $d$  using Baby-step Giant-step algorithm.

$d$  is not only fundamental discriminant.

Input number  $d$  must be int, long or **Integer**.

### 1.1.7 class\_group\_bsgs

```
class_group_bsgs(d: integer, cl: integer, qin: list)
    → integer
```

Return the construction of the class group of order  $p^{exp}$  with the discriminant  $disc$ , where  $qin = [p, exp]$ .

Input number  $d$ ,  $cl$  must be int, long or **Integer**.

### Examples

```
>>> quad.class_formula(-1200, 100000)
12
>>> quad.class_number(-1200)
12
>>> quad.class_group(-1200)
(12, [ReducedQuadraticForm(1, 0, 300), ReducedQuadraticForm(3, 0, 100),
ReducedQuadraticForm(4, 0, 75), ReducedQuadraticForm(12, 0, 25),
ReducedQuadraticForm(7, 2, 43), ReducedQuadraticForm(7, -2, 43),
ReducedQuadraticForm(16, 4, 19), ReducedQuadraticForm(16, -4, 19),
ReducedQuadraticForm(13, 10, 25), ReducedQuadraticForm(13, -10, 25),
ReducedQuadraticForm(16, 12, 21), ReducedQuadraticForm(16, -12, 21)])
>>> quad.class_number_bsgs(-1200)
12L
>>> quad.class_group_bsgs(-1200, 12, [3, 1])
([ReducedQuadraticForm(16, -12, 21)], [[3L]])
>>> quad.class_group_bsgs(-1200, 12, [2, 2])
([ReducedQuadraticForm(12, 0, 25), ReducedQuadraticForm(4, 0, 75)],
[[2L], [2L, 0]])
```